The Economics of Vehicle Driving: A General Equilibrium Analysis in a Dynamic Two-Period Vintage Model

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Dedicated to

My Parents

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Abstract

My thesis aims to explore the relationship between public policies and vehicle driving from three aspects.

First, we examine two policy options for the government to address pollution externality caused by vehicle driving: gasoline taxes and clean vehicle subsidies towards clean technology. We introduce vintage vehicles into our model to measure the impact of policies on households' vehicle driving choices. We show that all policies are effective in reducing pollution and improving the environmental quality. However, they have distinctively different distributional impact on the production side and social welfare.

Second, we derive the optimal environmental tax structure in the presence of externalities caused by vehicle driving in the first-best scenario. Analytical results show that the optimal gasoline taxes are composed of two opposing factors and depend on the household's preferences for environmental factors. Our calibration based on the U.S. economy shows that the optimal gasoline taxes should be higher for old cars while the optimal road taxes should be higher for new cars.

Third, we formulate the optimal environmental tax structure in the presence of other distortionary taxes. We find that the optimal environmental taxes constitute both the efficiency part and the Pigovian part. Optimal taxes depend not only on the household's preferences for the environmental factors but also on the degree of complementarities with normal consumption goods.

Declaration

The work in this thesis is based on research carried out at the Durham University Business School. No part of this thesis has been submitted elsewhere for any other degree or qualification and it is all my own work unless referenced to the contrary in the text.

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Chapter 1

Introduction

Among all consumer products, few are regulated and taxed more broadly than vehicles. Vehicle driving is the main stated reason for urban air pollution and congestion. Vehicles are the main culprits of urban air pollution, especially in developing countries(Jha and Whalley, 2015). Poor air quality is a major global problem, with outdoor air pollution causing more than 3.3 million annual premature deaths and many more associated cases of illness (Gately et al., 2017). Mobile sources are responsible for a large fraction of air pollutant emissions in the United States. In 2012, more than 75% of carbon monoxide (CO), and 60% of nitrogen oxides (NO_x) were emitted from on- and off-road vehicles, while mobile sources in large urban areas accounted for as much as 90% of local CO emissions (EPA, 2011). The average emission level of new domestic vehicles is three to ten times higher in developing countries than that in developed countries due to lagging automotive manufacturing technology, poor fuel quality, poor vehicle exhaust control, and lenient laws controlling vehicle emissions (He et al., 2002). Apart from environmental concerns, transport sector, which has been growing rapidly in the past decades has made urban traffic jams worse and poses a large challenge to public policy making in terms of oil security. Global vehicle ownership level has increased year on year in the last decade and accordingly the amount of crude oil consumption by the transport sector (IEA 2018[1]).

The 2019 Urban Mobility Report stated that congestion wastes a massive amount of time and fuel and creates more uncertainty for travellers and fright. In 2017, 8.8 billion hours of extra time were spent on roads and 3.3 billion gallons of fuel were wasted. An average auto commuter spent an extra 54 hours travelling and wasted 21 gallons of fuel in 2017.

With surging vehicles on roads, many countries find themselves more dependent on imported oil than at any time in history. Transport sector plays a more and more important role in the energy system, oil demand and CO_2 emissions. The United States is the largest economy nowadays and emitted 17.5% of the world's total CO_2 in 2012. USA's transport sector consumes 27.9% of the total final energy consumption (Zhang et al., 2016). In 1993, China became a net oil-importing country and the amount of oil imported by China in 2000 reached 70 million tons, which took up about 30% of that year's total oil consumption. The major reason of this increase can be attributed to the rapid growth of the transportation sector, particularly motor vehicles (He et al., 2005).

Both the public and governments are concerned about the perceived economic,

[1] The International Energy Agency's first Global Energy and CO2 Status Report (IEA 2018) provides a snapshot of recent global trends and developments across fuels, renewable sources, energy efficiency and carbon emissions from 2006 to 2017.

environmental and security vulnerabilities arising from vehicle driving and fuel consumption. Over the past decades, remarkable advances have been made to address externalities and curb fuel consumption. Vehicles are subject to regulations concerning pollution, safety and fuel economy. Among all kinds of policies that have been implemented, taxation has been widely applied in many countries. Levying tax on transportation fuel has been advocated to reduce pollution and conserve crude oil. It is also expected by many government officials that by levying tax on fuel, consumers would switch to public transport which is helpful to improve traffic situation. However, Parry et al. (2007) argues that, instead of using fuel tax, which is a very blunt instrument for alleviating traffic congestion, the ideal strategy should be a road-specific congestion toll. Apart from taxation, many government either tried to offer fiscal incentives to consumers to encourage them to purchase more fuel-efficient vehicles or to provide vehicle producers with subsidies to improve the fuel efficiency levels of the newly produced cars.

Although fiscal policies have been widely used in transport sector to address externalities and conserve energy, the mechanisms and interrelations behind these policies have never been thoroughly examined. The essential purpose of this thesis is to find out how different environmental taxes affect the economy, environment and social welfare. We have examined the variations and distributional effects when policy changes. We also derived the optimal environmental taxation in the presence of other taxes.

In this chapter, we illustrate the main literature that has been discussed in this thesis. In section 1.1, we introduce the global fuel consumption status and the externalities caused by vehicle driving. In section 1.2, we present how vehicle-driving related factors are modelled in the previous literature. Finally, section 1.3 provides the road map for this thesis.

1.1 Fuel consumption and externalities from vehicle driving

Gasoline has played an important role in accelerating the world's economy. The consumption of gasoline is closely related to the world's economy development status. Figure 1.1[2] shows the increasing trend of global oil consumption from 19906 to 2017. It is clear that Gas/Diesel consumption has been increasing over the past two decades at a steady growth rate.

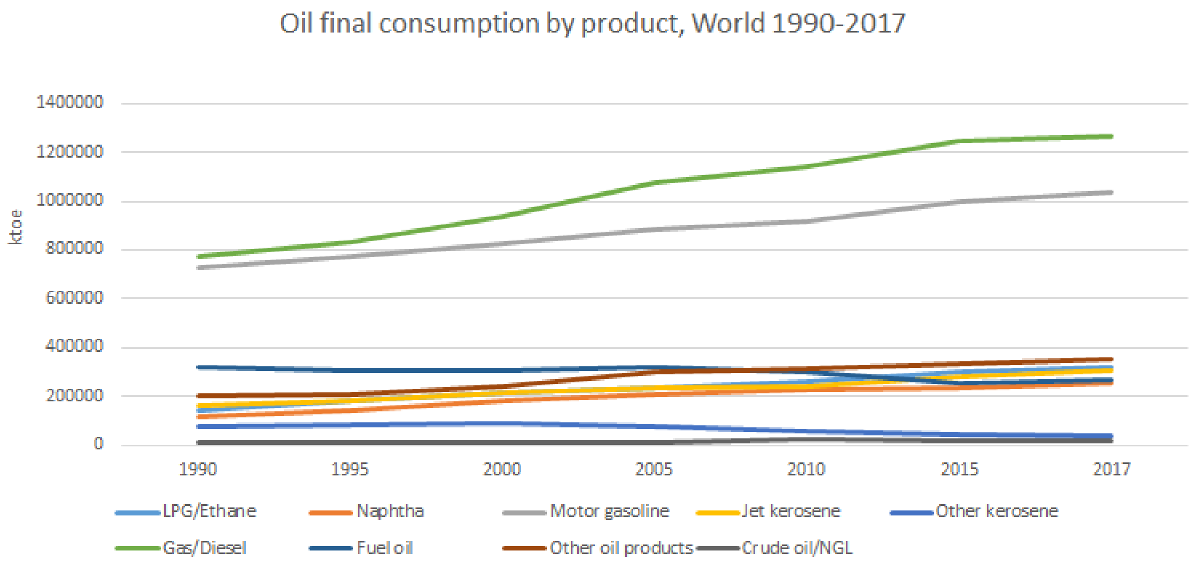

Figure 1.1: Oil final consumption by product, World 1996-2017

A major cause of the increase in oil consumption can be attributed to the rapid

[2] Data collected from the International Energy Agency (IEA) oil information 2018.

growth of the transportation sector. Road transportation has become increasingly important in the urbanization process. As a result, road transportation consumes a large amount of oil and leads to a large amount of carbon dioxide (CO_2) emission. It is estimated that the road transportation system accounted for 61% of oil consumption and 70% of CO_2 emissions of the whole transport sector (He et al., 2005). Apart from carbon emissions, another significant externality caused by vehicle driving is air pollution. Gasoline vehicles emit carbon monoxide (CO), nitrogen oxides (NO_x), and hydrocarbons (HC). CO leads to a reduction of oxygen in the bloodstream and causes breathing difficulty and cardiovascular effects while HC and NO_x react to sunlight to form ozone (the main component of smog) that affects pulmonary function of children and reduces visibility. More importantly, NO_x and HC also react to form particulate matter. Fine particles (PM2.5) are small enough to reach lung tissue and a causal relation between particulate exposure and mortality was documented by several studies[3]. All these effluents have posed great threat to human health.

Another arresting externality caused by gasoline vehicles is traffic congestion. Between 1980 and 2003, urban VMT (Vehicle Miles of Travel) in the United States has increased by 111%, against an increase in lane-mile capacity of only 51%[4]. Annual urban congestion delays increased from 16 to 47 hours per driver, while the national cost of wasted time from congestion increased from \$12.5 to \$63 billion (Lomax and Schrank, 2005).

[3] See Dockery et al. (1993) and Schwartz (1994).

[4] Data from U.S. Department of Transportation, Bureau of Transportation Statistics 2005.

1.2 The economy of vehicle

This section provides reviews on how previous literature modelled vehicle driving service and vehicle attributes.

1.2.1 Driving service

Vehicle driving contributes to many externalities and lots of the externality-related problems are solved through individual choice problems in economic literature. The standard procedure is to put vehicle-related factors into economic agent's utility function separately from consumption and leisure. To produce vehicle miles of travel, both vehicle and gasoline are needed.

Vehicle, as a type of capital, and gasoline could enter the household's utility function separately. Bento et al. (2009) assume households obtain utility from car ownership and utility depends on characteristics of the automobile as well as vehicle miles of travel. In their model, if the households have car endowment, they need to choose whether to hold the car or to scrape it; if the households relinquish the car, they also need to decide whether to purchase a new one or not. The representative household's utility is expressed by:

$$U_{ij} = U_{ij}(z_j, M_i, x_i), \tag{1.2.1}$$

where z_j is a vector of qualities of car j. M_i and x_i, respectively, refer to household i's vehicle miles of travel and its consumption of normal good.

Putting vehicle characters and vehicle miles of travel separately in the utility function, the qualities of vehicles are depicted more delicately. However, vehicle miles of travel is closely related to both the vehicle type and the amount of gasoline consumed. Thus, modelling vehicle driving in this way fails at capturing that interrelation.

Parry and Small (2005) consider a static model in a closed economy to derive optimal fuel tax formula, taking vehicle-related externalities into consideration. In their model, vehicle type and gasoline consumption together determine vehicle miles of travel. The representative household's utility function takes the following form:

$$U = U(\Psi(C, M, T, G), N) - \phi(P) - \delta(A), \tag{1.2.2}$$

where C denotes the quantity of a numeraire consumption good, M vehicle miles of travel, T the time spent on driving, G government spending, N leisure, P the quantity of pollution and A the severity-adjusted traffic accidents.

It is clear that the utility function has been refined to better present vehicle-related factors. Especially, they define vehicle miles of travel M as:

$$M = M(F, H), \tag{1.2.3}$$

where F denotes gasoline consumption and H represents a monetary measure of other driving costs which depends on vehicle prices and attributes.

This function embeds the inner substitution effect between gasoline consumption and vehicle attributes. When the price of gasoline increases, drivers either drive less (lower H) or switch to more fuel-efficient ones which increase H. In this way, the interrelation between gasoline consumption and vehicle attributes is successfully captured. However, they do not specify an exact function form to illustrate this relationship.

1.2.2 Fuel efficiency and vintage

Given the fact that both vehicle and gasoline are needed in order to produce mileage of travel, it is inevitable to model car attributes. Among all the attributes, fuel efficiency is the main cause for different vehicle mileage of travel given the same amount of gasoline. Therefore, how to model fuel efficiency is rudimentary for the model setting. Previous research have tried different ways to model fuel efficiency.

In the foregoing discussion, Parry and Small (2005) put car attributes and gasoline consumption together to produce vehicle miles of travel (Eq. 1.2.3). Although the interaction between F and H is not stated explicitly, this function allows for a non-proportional relation between gasoline consumption and vehicle miles of travel. However, what the exact relation is calls for further assumption and explanations.

Fuel efficiency standards have been applied worldwide as a regulatory mechanism to address externalities and preserve oil. In the wake of 1973 oil crisis, the corporate average fuel economy (CAFE) was put into practice in the United States. These standards impose a limit on the average fuel economy of the vehicles sold by a particular company each year, with separate limits for passenger cars and light duty trucks (Jacobsen, 2013). Given that the CAFE standards state the fuel economy in terms of miles-per-gallon, lots of research use miles-per-gallon to proxy fuel efficiency when evaluating the policy empirically. However, this method has disadvantage in analytical study: it ignores the interrelation between fuel efficiency and gasoline consumption.

Wei (2013) comes up with a model where she uses production function to model the relation between fuel efficiency, gasoline consumption and vehicle miles of travel.

At time $t-j$, the representative household chooses fuel efficiency by choosing k_{t-j} which represents both the transportation capital configured in the vehicle and the capital-gasoline ratio at full capacity. Once household made the decision, an idiosyncratic productivity term, ζ_i , is revealed for each vehicle i. Once settled, ζ_i and k_{t-j} will not change during the life span of the vehicle. Denoting the gasoline use by $O_{i,t,j}$, vehicles miles of travel in period t by vehicle i produced at $t-j$ is:

$$M_{i,t,j} = \zeta_i k_{t-j}^{\alpha} O_{i,t,j}. \tag{1.2.4}$$

Wei (2013) uses putty-clay production technology to differentiate vehicles embedding different fuel efficiency levels and that is vehicles with different vintages. Putty-clay technology, originally introduced by Johansen (1959), provides an alternative description of production and capital accumulation that breaks the tight restrictions on short-run production possibilities imposed by Cobb-Douglas technology. He builds up a natural framework for examining issues related to irreversible investment. With putty-clay capital, the ex-ante production technology allows for substitution between capital and labour, but once the capital good is installed, the technology is Leontief with productivity determined by the embodied level of vintage technology and the ex-post fixed choice of capital intensity (Gilchrist and Williams, 2000). In producing vehicle miles of travel, the producing inputs are transportation capital and gasoline. Before investing in transportation capital, the production technology is considered to be in Cobb-Douglas form with constant return to scale. However, once the configuration is set, transportation capital and gasoline could not substitute each other as in the normal Cobb-Douglas production function. The only production input is the transportation capital-gasoline ratio.

Solow et al. (1960a), Cooley et al. (1997) and Jovanovic (1998) look into other ways to interpret vintage capital. They argue that the latest technology is only incorporated in the latest capital, while old capital still uses the technology from the time it was produced. Therefore, an economy where old capital embedded with old technology while new capital with latest technology is more realistic. In Solow (1962), capital has a fixed lifetime and the amount of labour allocated to given unit of capital is fixed at the time it is introduced.

Consider a representative plant owning capital of vintage i. This plant can choose to produce either consumption goods or capital goods. Consumption goods production function is:

$$c_i = k_i^{\alpha} l_i^{\beta}, \tag{1.2.5}$$

where $0 \leq \alpha, \beta, \alpha + \beta \leq 1$.

c_i is the consumption goods output, and k_i, l_i denote capital of vintage i and unskilled labour. Instead of fixing production input ratio, they use capital heterogeneity to model capitals embedding different technologies. Putty clay is of great value in modelling the relationship of vehicle miles of travel.

1.3 A road map for this thesis

In this thesis, we aim to explore the relationship between vehicle driving and public policies from three different angles: how different policies affect vehicle driving decision making, the optimal environmental tax structure in the first best, and the optimal environmental taxes in the presence of other distortionary taxes.

In Chapter 2, we examined the impact of two policy options on vehicle driving, environmental quality, the economy and social welfare: 1) fuel tax, and 2) clean vehicle subsidies. This model is characterised by two production sectors, namely, the general production sector producing consumption goods and the vehicle production sector producing vehicles of different vintages (new and old). In line with empirical findings, our analytical results illustrate that households prefer new cars rather than old ones so that more gasoline is consumed by new cars. Our simulation on the U.S. economy shows that all three policy options are efficient in reducing pollution. However, they have distinctly different distributional effects on the economy and social welfare.

In Chapter 3, we explored the structure of optimal environmental taxes in the presence of driving externalities (pollution and congestion) by extending the model developed in Chapter 2. We find that the optimal gasoline taxes are determined by two opposing forces caused by gasoline consumption: marginal cost of pollution and marginal cost of congestion. Optimal road taxes formulas show that the tax rates in the long run depend on the gasoline consumption ratio between the new cars and the old cars. Our calibration on the U.S. economy shows that the optimal levels of environmental taxes are affected by the households' preferences on the environmental factors. Moreover, to match with real life scenario, we derived the optimal uniform gasoline tax and examine the impact on the optimal road taxes. We find that the cost of long-run social welfare increases slightly when uniform gasoline tax is charged. However, the difference is not substantial.

In Chapter 4, we examined the optimal environmental tax structure when other distortionary taxes are considered. We find that the additive property between the

Pigovian element and the efficiency element proposed by Sandmo (1975) is retained in our model. And the optimal tax formulations are determined by the degree of complementarity with normal consumption goods.

Finally, Chapter 5 provides summaries of each chapters and future work plans.

Chapter 2

Fuel Tax, Clean Vehicle Subsidies and Earmarking Policy: A General Equilibrium Analysis in a Dynamic Two-Period Vintage Model

In this chapter, we develop a dynamic general equilibrium infinite-horizon model with physical capital and vehicles, where vehicles are of two vintages (new and old), and investigate the impact of fuel taxes and clean technology subsidies to fuel efficiency production on driving behaviour, vehicle production, fuel consumption, environmental quality and welfare. We first show that, because of new cars are embedded with higher fuel efficiency, households proportionally drive new cars for longer distances (or more often) than old cars. This leads to new cars consuming more fuel in equilibrium. Subsequently we explore the effects of the policy options numerically. Computation results of the steady states show that all policies reduce overall fuel consumption which leads to a lower level of pollution and thus enhance environmental quality. However, clean technology subsidies distort resource allocation in the vehicle production sector (causing production inefficiency) which in turn leads to a decrease in the general consumption, fuel consumption (through a decrease in fuel import) and leisure (through the income effect). Social welfare depends on the subsidy level. Low subsidies increase welfare very rapidly while higher levels decrease it. Because of the overall increase in consumption and environmental quality, social welfare improves.

2.1 Introduction

Vehicles are the main culprit of urban air pollution, especially in developing countries (Small and Kazimi, 1995). Global vehicle ownership levels has increased year on year in the last decade and accordingly the amount of crude oil consumption by the transport sector (IEA 2018 [1]). In addition to increased oil consumption, emissions from transport sector can be attributed to missed opportunities for improving energy efficiency and lenient pollution regulations, especially in developing countries. The average emission levels of new domestic vehicles are 3-10 times higher in developing

[1]The International Energy Agency's first Global Energy and CO2 Status Report (IEA 2018) provides a snapshot of recent global trends and developments across fuels, renewable sources, energy efficiency and carbon emissions from 2006 to 2017.

countries than that in developed countries (He et al., 2002).

There has been heated discussions in policy circles on reducing fuel consumption, improving fuel efficiency, providing incentives for clean technology and therefore addressing the pollution externality caused by vehicle driving. Many countries have attempted to address these problems by implementing fuel efficiency standards[2], and market-based mechanisms of pollution controls such as fuel taxes and subsidies (towards both consumers and manufacturers). Fuel taxes made its first appearance in early 1900s as a way to raise government revenue and are widely used by many countries nowadays (See OECD (2018)). Energy efficiency subsidies are a more recent government policy having appeared in the early 2000s. For example, in the US, the Energy Policy Act of 2005 provided for a maximum of $3400 tax credit towards hybrid electric vehicle (HEV) purchase between 2006 and 2010 (Hao et al., 2014). Firms were also given incentives to produce clean vehicle. The Partnership for a New Generation of Vehicles, formed in 1993, was a project conducted between the U.S. government and the three major domestic auto corporations, aimed at bringing fuel-efficient vehicles to the market (McCosh, 1994). During Obama administration, the U.S. government pledged $2.4 billion in federal grants to support the development of next generation electric vehicles and batteries. The funds were allocated to manufacturers towards three main streams: 1) the production of highly efficient batteries and their components; 2) the production of other components needed for

[2] For example, Corporate average fuel economy (CAFE) standards are enacted by the United States in 1975. European Union has entered into a series of voluntary agreements called the European Union Automotive Fuel Economy Policy.

electric vehicles; 3) the demonstration and evaluation of plug-in hybrids and other electric infrastructure concepts[3].Recently the attention has shifted from looking at those instruments in isolation towards policies that combine them (Tanishita et al., 2003). Earmarking the revenues from the gasoline taxes towards subsidies for improving fuel efficiency is also gaining support in political circles.

This paper aims at providing a detailed theoretical framework to assess the impact on fuel consumption, fuel efficiency, pollution, environmental quality and welfare under two different policy options: 1) fuel tax on households' fuel consumption and 2) subsidy towards clean technology, which targets specifically at the engine of the vehicles, in vehicle production sector.

While the empirical literature on estimating the economic and environmental impact of policies is vast[4], the theoretical literature analyzing fuel policies is quite limited.

Parry and Small (2005) sets up a structural static model to determine the optimal fuel tax in the presence of externalities caused by driving where revenue from fuel tax is used towards reducing the households labour income tax. They show that the current fuel tax rate is too low in the United States and too high in the United Kingdom.

Wei (2013) constructs a dynamic vintage model to assess the economic and environmental impact of increasing fuel taxes (with revenues being recycled through

[3] Like truck stop charging station, electric rail, and training for technicians to build and repair electric vehicles (green jobs).

[4] Parry et al. (2007) reviews the empirical literature on automobiles externalities and policies to address them.

lump-sum transfers) and tightening fuel efficiency standards. She shows that both policy instruments are successful in the long run in reducing fuel consumption, but they are different in their transmission channels and may have different economic and environmental impact.

Our model is novel in several aspects. First, differently from Parry and Small (2005), we develop a framework where dynamic relationships are present to capture the long-run nature of pollution and capital accumulation. A dynamic model is useful to interpret pollution issues as those generally accumulate over time and also affect environmental quality over time. Any policies addressing pollution issues will also have long-run effects both on the environment and social welfare. Furthermore, we do not analyse optimal policies but focus on fuel policy reforms.

Differently from Wei (2013) who adopts a vintage model with putty-clay technology to model households' driving decisions, we introduce vintage vehicles using capital heterogeneity. Solow et al. (1960a) and Cooley et al. (1997) point out that the latest technology is only incorporated into the latest capital, while old capital still uses the technology from the time it was produced. Wei (2013) uses putty-clay technology with Leontief production possibilities to model vehicle mileage of travel where the ratio of vehicle capital to energy consumption is fixed *ex post* production. Vehicle capital, however, is special in that it could generate mileage of travel given any amount of fuel pumped in. Leontief possibilities thus do not match with vehicle features. We therefore adopt capital heterogeneity to model mileage of travel.

Furthermore we offer a novel way of modelling vehicle capital and fuel efficiency. Previous theoretical literature (e.g.Wei (2013) and Parry and Small (2005)) has assumed that all components of vehicle capital are indistinguishably linked to fuel efficiency. We expand this framework to model two distinct attributes of vehicle capital, one being the size of the vehicle and the other embedding fuel efficiency and clean technology. An example of an attribute reflecting the latter dimension is a more fuel-efficient engine which not only results in more miles per gallon (mpg) but also mitigate the polluting emissions (e.g. a hybrid electric motor). This specification allows us to capture the firm's choice in the quality-quantity dimension (whether to produce bigger-sized cars or more fuel-efficient and cleaner engines) and ultimately the resulting overall fuel efficiency of the vehicle (the end product). In addition, it enables us to investigate the role of government subsidies in influencing production of more fuel-efficient and cleaner engines.

We summarize the results as follows. First, in terms of driving choices, households purchase more fuel for new cars than old cars. Households also prefer to use new cars more often than old cars. Second, simulation results show that fuel consumption and pollution levels decrease under all policies.

Levying fuel tax does not improve the overall fuel efficiency (mpg) of the vehicles and also barely changes output. It alleviates pollution which in turns enhances environmental quality and eventually improves social welfare.

Providing subsidies, instead, leads to more resources allocated to the production of more fuel-efficient and cleaner engines, which results in higher capital accumulation and labour supply in the production sector. The overall fuel efficiency does not change significantly as producers substitute away from the size attribute towards the fuel-efficient attribute. As subsidy rate increases, social welfare first improves and then plunges when production inefficiencies kick in.

The structure of the paper is as follows. Section 2.2 describes the model and its dynamics. Section 2.3 presents the steady-state analytical results. Section 3.2 presents the benchmark calibration. Section 2.5 examines the economic impact of the three policies. Section 4.6 concludes.

2.2 The model

This section describes a decentralized economy including firms, households and the government. Section 2.2.1 presents the production technology and the profit-maximizing problems for the firms. Households' problem modelling consumption, driving and other services is discussed in section 2.2.2 where we also explain how capital heterogeneity is applied to model driving services. Government's policy options are discussed in section 2.2.3.

2.2.1 Firms

There are two production sectors in the economy: the general production sector G and the vehicle production sector F.

General production sector

At each period t, firms hire labour l_t^g and capital k_t^g at the rate of w_t^g and r_t^g from the households to produce final output which can be used for consumption, capital accumulation and fuel import. The generated profits π_t^g goes to the households.

The final good, G, is produced with constant-return-to-scale technology:

$$G(k_t^g, l_t^g) = A_1 (k_t^g)^{\alpha_1} (l_t^g)^{1-\alpha_1}, \tag{2.2.1}$$

with resource constraint:

$$G(k_t^g, l_t^g) = c_t + k_{t+1} - (1-\epsilon_k)k_t + p_t(g_{t,1} + g_{t,2}), \tag{2.2.2}$$

where c_t denotes consumption, ϵ_k measures capital depreciation and p_t is fuel price assumed to be fixed in the world market[5]. $g_{t,1}$ represents fuel consumed by new cars and $g_{t,2}$ old cars. k_t denotes the total capital at time t.

Firms maximize profits π_t^g at each time period with respect to the amount of capital k_t^g and labour l_t^g they hire[6].

$$\max_{k_t^g, l_t^g} \pi_t^g = G(k_t^g, l_t^g) - w_t^g l_t^g - r_t^g k_t^g. \tag{2.2.3}$$

The corresponding first order conditions are:

$$r_t^g = G_{k_t^g}, \tag{2.2.4}$$

$$w_t^g = G_{l_t^g}. \tag{2.2.5}$$

Vehicle production sector

Vehicle, as a type of capital good, is made up of two attributes, the chassis of the car a and the fuel efficiency component δ (the car engine power). Those two

[5]We assume that the gasoline consumption in this open economy depends on import and the demand will not affect the world oil price.

[6]We model the decentralised economy as the households bear the depreciation costs. Another equivalent way would be firms bear the depreciation which would change the factor price in the equilibrium accordingly.

components are produced separately but must be sold as a combined product. Fuel efficiency, embedding clean technology, is the crucial part in mitigating air pollution (see Eq.2.2.21).

Firms in the vehicle production sector hire labour l_t^a and capital k_t^a at the rate of w_t^a and r_t^a to produce vehicle capital a_t and fuel efficiency δ_t [7]. Firms sell the final vehicle product $a_t\delta_t$ to households at the price q_t^a. The resource constraint reads:

$$F(k_t^a, l_t^a) = a_t + \mu\delta_t{}^{8}, \tag{2.2.6}$$

where μ is the marginal rate of transformation between a and δ and $\mu > 0$.

The problem facing the firms in this sector is:

$$\max_{k_t^a, l_t^a, \delta_t} \pi_t^a = q_t^a(a_t\delta_t) - r_t^a k_t^a - w_t^a l_t^a + (s_t\delta_t), \tag{2.2.7}$$

where $s_t\delta_t$ appears when government adopts the policy of providing subsidy towards clean technology (higher δ) and s_t denotes the subsidy rate.

We assume Cobb-Douglas technology in labour and capital:

$$F(k_t^a, l_t^a) = A_2(k_t^a)^{\alpha_2}(l_t^a)^{\frac{1}{2}-\alpha_2}. \tag{2.2.8}$$

Notice that vehicle production, $a\delta$, is constant-return-to-scale in capital k_t^a and l_t^a (i.e. doubling k_t^a and l_t^a, will double $a\delta$)[9].

[7] Differently from Wei (2013), we have disentangled the vehicle into the vehicle capital and engine fuel efficiency so that we could model specifically the effect of subsidies.

[8] The resource available in vehicle production sector (Eq.2.2.8) can be allocated to either produce more vehicle capital (a) or more fuel efficiency (δ). It shows the trade-off between quality and quantity. If firms decide to put more resource to produce vehicle capital a, then the fuel efficiency δ embedded in the vehicle will be lower which means that vehicles are less efficient in producing mileage of travel.

[9] We specifically assume that the power adds up to $\frac{1}{2}$. If there is no externalities in the economy,

The first-order conditions for the vehicle producer are:

$$r_t^a = q_t^a \delta_t F_{k_t^a}, \tag{2.2.9}$$

$$w_t^a = q_t^a \delta_t F_{l_t^a}, \tag{2.2.10}$$

$$q_t^a [F(k_t^a, l_t^a) - \mu\delta_t] - \mu q_t^a \delta_t + (s_t) = 0. \tag{2.2.11}$$

Equilibrium conditions in production

Market clearing implies:

$$k_t^a + k_t^g = k_t, \tag{2.2.12}$$

$$l_t^g + l_t^a = l_t, \tag{2.2.13}$$

where k_t denotes the total capital and l_t the total labour at time period t and

$$w_t^a = w_t^g = w_t, \tag{2.2.14}$$

$$r_t^a = r_t^g = r_t. \tag{2.2.15}$$

2.2.2 Households

Preference

Many identical infinitely-lived households face log preferences for consumption c_t, driving service M_t, leisure $1 - l_t$ and environmental quality N_t.

$$U(c_t, M_t, 1 - l_t, N_t) = \phi_1 \log c_t + \phi_2 \log M_t + (1 - \phi_1 - \phi_2) \log (1 - l_t) + \phi_3 \log N_t. \tag{2.2.16}$$

Equation 2.2.11 become $q_t^a\,(F - \mu\delta_t - \mu\delta_t)$. In order for the final production to be constant return to scale, we will have $\frac{q_t^a}{\mu}(\frac{F}{2})^2 - r_t^a k_t^a - w_t^a l_t^a = 0$. Thus,to make $(\frac{F}{2})^2$ constant return to scale, F has to be diminishing return to scale and the power has to sum up to $\frac{1}{2}$.

Production of driving services

There are two types of vehicles in the market: new cars and old cars. We follow Solow et al. (1960b) and Cooley et al. (1997) to model vintage capital with "putty-clay" technology. After production, the technology embedded in the vehicle will not change, which implies that the mileage of travel over one unit of fuel consumed is fixed for different vehicle vintages. Vehicles need one period of configuration and will be used by households for two periods before getting scraped. New cars are produced at time period $t-1$. Old cars are produced at time period $t-2$ and are also subject to depreciation $1-\rho$ from already being used for a time period. Following Wei (2013), mileage of travel produced by new cars $m_{t,1}$ and old cars $m_{t,2}$ are:

$$m_{t,1} = (a_{t-1}\delta_{t-1})^{\gamma} g_{t,1}, \tag{2.2.17}$$

$$m_{t,2} = (\rho a_{t-2}\delta_{t-2})^{\gamma} g_{t,2}, \tag{2.2.18}$$

where $0 < \rho < 1$ and $0 < \gamma < 1$. γ measures the production technology embedded in the vehicle. If γ becomes higher, given the same amount of gasoline, more mileage of travel will be produced.

The representative household owns both new cars and old cars. Driving service M_t at each time period is composed of mileage of travel produced by new cars $m_{t,1}$ and old cars $m_{t,2}$:

$$M_t = (m_{t,1}^{\sigma} + m_{t,2}^{\sigma})^{\frac{1}{\sigma}}, \tag{2.2.19}$$

where $0 < \sigma < 1$ and it measures the price elasticity of demand.

We set the preference for M_t following Grossman and Helpman (1991) to guarantee that household exhibits preference for variety over quantity, which means that household always prefers to use both types of cars instead of just using new cars.

Environmental quality

Environmental quality is modelled as a type of renewable resource. The quality of the environment, N, represents the stock of natural capital and accumulates based on the regenerating ability of nature while depreciates due to pollution P. N evolves over time according to the following function based on Bovenberg and Smulders (1995):

$$N_{t+1} - N_t = E(N_t) - P_t, \tag{2.2.20}$$

where $E(N_t)$ represents the nature's assimilating ability or ecological services produced by nature, that is the amount of pollution that can be assimilated without a change in the environmental quality. We could also interpret Eq.2.2.20 as that changes to the environmental quality and pollution are two rival users of ecological services. Nature's assimilating ability $E(N_t)$ takes the function form:

$$E(N_t) = \phi - \epsilon N_t,$$

where ϕ denotes the original state and ϵ represents the nature's rate of assimilating pollutants $(0 < \epsilon < 1)$.

Pollution

Our specification of pollution is based on Selden and Song (1995): pollution P_t is caused by the consumption of fuel $(g_{t,1}, g_{t,2})$ but mitigated by vehicles' fuel-efficiency conditions $(\delta_{t-1}, \delta_{t-2})$, with $\partial P/\partial g > 0$, $\partial P/\partial \delta < 0$, $\partial^2 P/\partial g^2 = 0$ and $\partial^2 P/\partial \delta^2 > 0$.

$$P_t = \frac{g_{t,1}}{\delta_{t-1}^{\mu_1}} + \frac{g_{t,2}}{\delta_{t-2}^{\mu_2}}, \tag{2.2.21}$$

where parameters μ_1 and μ_2 measure the ability of mitigating pollution by different vintages of cars and $\mu_1 \geqslant \mu_2$.

Household's problem

Each period, the representative household supplies labour l_t and capital k_t to firms and receives the profits generated in both sectors (π_t^g and π_t^a). Household purchases consumption goods, fuel, new vehicles and invest.

Household maximizes its life-long utility:

$$\max_{c_t, g_{t,1}, g_{t,2}, a_t \delta_t, l_t} \sum_{s=0}^{\infty} \beta^{t+s} U(c_t, M_t, 1 - l_t, N_t), \tag{2.2.22}$$

subject to the restriction:

$$\pi_t^a + \pi_t^g + w_t l_t + r_t k_t + T_t = (p_t + \tau_t)(g_{t,1} + g_{t,2}) + k_{t+1} - (1 - \epsilon_k) k_t + c_t + q_t^a (a_t \delta_t). \tag{2.2.23}$$

The household takes the environmental quality N_t as given. τ_t is the unit tax levied by the government on the consumption of fuel if government were to adopt fuel tax policy. T_t represents the lump-sum tax (negative) if government were to implement production subsides towards clean technology . It becomes lump-sum transfer (positive) if government were to levy tax on fuel consumption. Notice that vehicle price q_t^a clears the market for household and vehicle production sector.

The optimality conditions are derived in Appendix 2.7.1.

2.2.3 Government

Government has two policy options: 1) Levy tax τ_t on household's purchase of fuel, 2) Subsidize firms' production of more fuel efficient engines (s_t) and 3) earmarking the revenues from the gasoline taxes towards subsidies for improving fuel efficiency.

The first two policy options will always hold through lump-sum transfer by government. Under the earmarking policy, the government constraint reads:

$$s_t\delta_t = \tau_t(g_{t,1} + g_{t,2}). \tag{2.2.24}$$

2.3 Equilibrium

In this section, we characterize the steady-state solutions of the model and derive long-run fuel consumption and households' driving decisions[10]. A competitive equilibrium needs to be defined first.Take all the prices as given, 1) households maximise Eq.2.2.22 subject to Eq.2.2.23; 2)representative goods producer maximises profits according to Eq.2.2.3; 3) representative vehicle producer maximises profits according to Eq.2.2.7 and 4) markets clear according to Eq.2.2.2 and Eq.2.2.23.

Proposition 1. *The long-run ratio of fuel consumption between the new cars and the old cars is given by:*

$$\frac{g_{1,ss}}{g_{2,ss}} = \rho^{\frac{\gamma\sigma}{\sigma-1}}. \tag{2.3.25}$$

Here the subscript ss represents the steady state.

Proposition 1 characterizes the fuel consumption ratio between new cars and old cars. The steady-state fuel consumption ratio does not depend on the policy. It only depends on the depreciation rate of vehicle $1-\rho$, mileage production technology γ and driving service preference σ. In the long run, new cars in total will consume

[10]Given the gasoline consumption ratio and miles-of-travel ratio, we can also conclude that there is no rebound effect in this model.

more fuel compared to old cars despite being more fuel-efficient[11].

Proof. See Appendix 2.7.2 □

Proposition 2. *Using the previous result, we can also obtain the mileage-of-travel ratio between the new cars and the old cars:*

$$\frac{m_{1,ss}}{m_{2,ss}} = \rho^{\frac{\gamma}{\sigma-1}}. \tag{2.3.26}$$

Proof. See Appendix 2.7.3 □

Proposition 2 characterizes the equilibrium solution to the mileage ratio among two types of vehicles. Overall, households prefer to use new cars more often than old cars given that new cars are more efficient in providing driving services.

In the next two sections, we calibrate the model and use the analytical closed-form solutions to characterize the paths of the key endogenous variables responding to different policy options.

2.4 Calibration

This section describes the benchmark calibration of the parameters. The values of parameter (shown in Table 2.1) are based on the comprehensive reviews of relevant literature, like Wei (2013), Parry and Small (2005) and Chen et al. (2006). There are four categories of parameters: the first relates to driving service and fuel usage.

[11] $1-\rho$ measures the depreciation of vehicle having been used for a period ($0 < \rho < 1$). γ measures the production level of fuel efficiency ($0 < \gamma < 1$) and σ measures the price elasticity of demand in driving service ($0 < \sigma < 1$).

The second is about production technology and the third specifies the preferences of the household. The forth category is about the environmental quality and pollution. The details of calibration can be found in Appendix 2.7.4.

Category	**Parameters Description**	**Notation**	**Value**
Driving Service	Vehicle leftover rate	ρ	0.9
	Vehicle preference	σ	0.5
	Mileage production technology	γ	0.42
Production Technology	Capital depreciation rate	ϵ_k	0.1
	Capital share in production	α_1,α_2	0.33/0.42
	Productivity level	A_1, A_2	1
	Marginal transformation rate	μ	1
	Fuel price	p_t	1.0872
Household Preference	Subjective discount rate	β	0.97
	Weight on consumption	ϕ_1	0.34
	Weight on driving	ϕ_2	0.05
	Weight on environmental quality	ϕ_3	1
Environmental Factor	The capacity of fuel efficiency	μ_1,μ_2	1
	Original state of environment	ϕ	0.25
	Natural purifying capacity	ϵ	0.1

Table 2.1: Benchmark Calibration

2.5 Comparative statics of policies

In this section, we use the calibrated model to examine the efficacy of the three policy options in addressing pollution and curbing fuel consumption: 1) fuel taxes on households' fuel consumption (τ_t); 2) subsidies towards clean technology (s_t) and 3) earmarking gasoline taxes revenue to subsidize the production of more fuel-efficient vehicles. We first examine the long-run paths of endogenous variables responding to increasing tax rates and subsidy rate. We then assess their long-run impact on environmental quality. Finally we examine whether the policies make the society better off or not.

2.5.1 The impact of policies on economy

In this section, we plot the equilibrium paths of key endogenous variables responding to increasing fuel prices and increasing subsidies.

Figure 2.1 shows the impact of increasing fuel tax on the economy. In the long run, capital accumulation, labour supply decreases slightly. Fuel efficiency and vehicle price rarely change. The direct impact of fuel tax is on fuel consumption. Increasing tax rate makes it more expensive to purchase fuel and households thus reduce their demand for fuel. Eq.2.3.25 characterizes the constant fuel consumption ratio between new cars and old cars. This is also observed in Fig.2.1 where fuel consumed by both new cars and old cars keep decreasing but the ratio stays unchanged. Moreover, households switch their demand from driving towards general consumption facing increasing fuel price. Pollution gets alleviated only due to the decreasing fuel consumption.

Figure 2.2 depicts the economy under subsidy policy option. As illustrated in our model, vehicles are produced by capital k^a and labour l^a. Increasing clean technology subsidies provides incentive for firms to allocate more resources to produce more

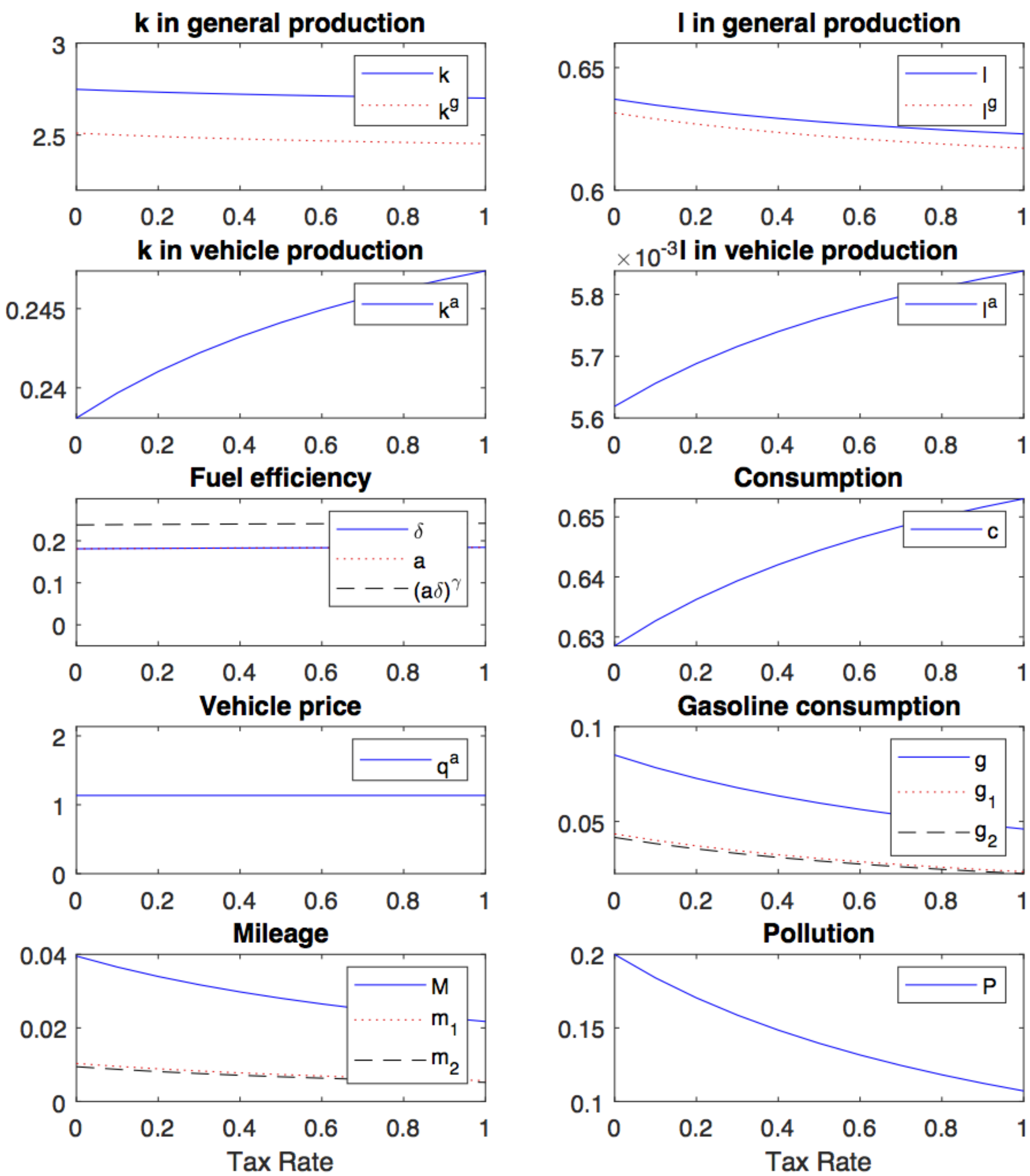

Figure 2.1: Comparative-static effects of implementing fuel tax τ alone.

fuel-efficient and cleaner engines (δ). Thus proportionally, δ takes heavier weight in the vehicle production sector. That explains the increment of δ and contraction of a. Overall, miles per gallon of a vehicle, measured by $(a\delta)^\gamma$ does not change significantly. Households, at the beginning, benefit from driving more as mpg of vehicles improve. After certain point, however, the product gap between a and δ is so big that the production inefficiency starts to kick in, which leads to households decrease

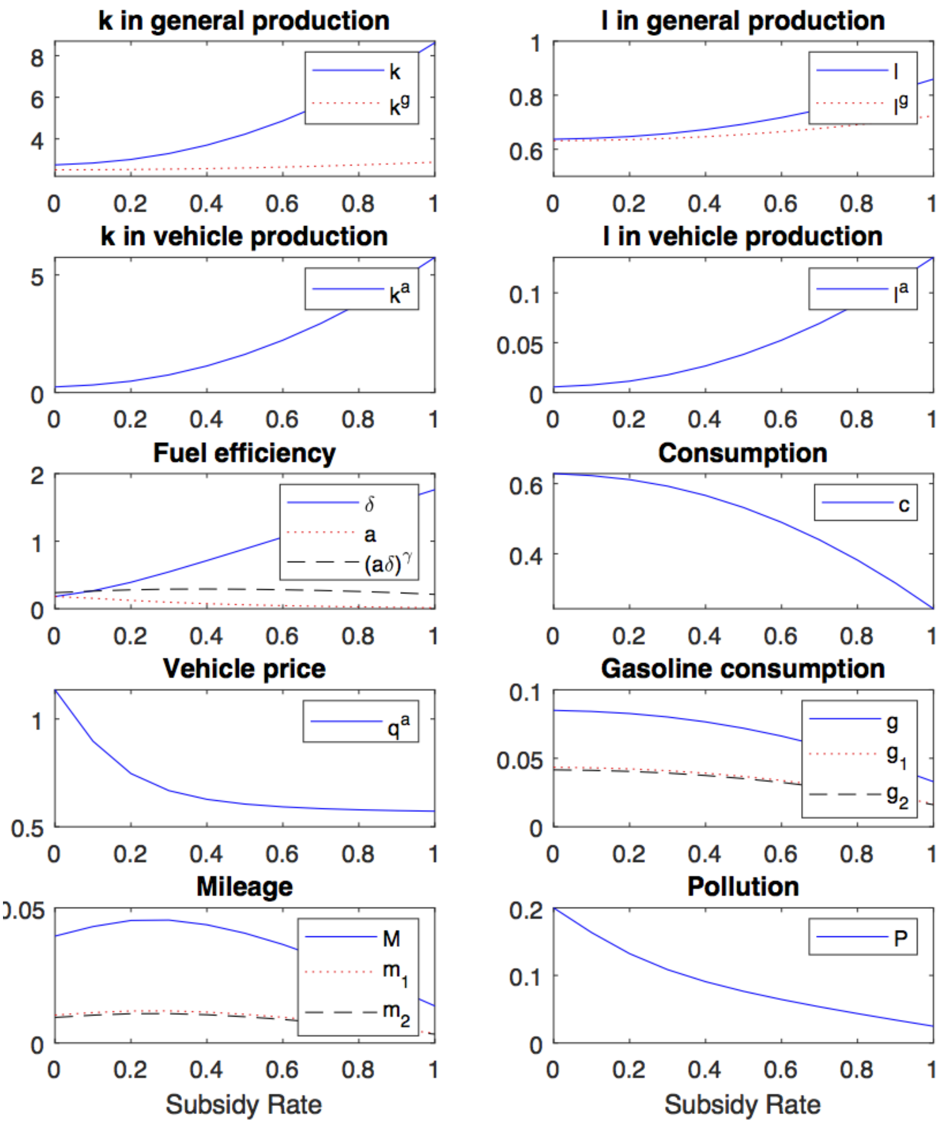

Figure 2.2: Comparative-static effects of implementing subsidy s alone.

their demand for driving. Production inefficiency also results in consumption and leisure monotonically decreasing in level of subsidy. Pollution level keeps dropping due to both the improvement of fuel efficiency (δ) and the decreasing consumption of fuel.

2.5.2 The impact of policies on the environment and welfare

The major goals of levying tax and subsidizing the environmentally-friendly engine production are to curb fuel consumption and to address the pollution issues caused by vehicle driving so as to improve social welfare. In this section, we use the calibrated model to examine the impact of policies on the environmental quality and welfare and also check the robustness of our model by conducting sensitivity analysis with different parameter values.

Environment

We investigate the long-run effect of the policies on the environmental quality. Environmental quality N is a stock value and its change depends on two opposing factors: nature's assimilating capacity and the pollution.

We measure different levels of long-run environmental quality when the environment assimilating ability differs (ϵ). Fig.2.3 depicts the effects of policies on the environmental quality under two policy options in turn: 1) fuel tax only and 2) subsidy towards clean technology. Environmental quality improves due to the decreasing pollution under all policies. A lower level of pollution (higher nature's assimilating capacity) means that fewer ecological services are needed to compensate for the adverse effect pollution has had on the environmental quality. Lower pollution in turn results in a higher level of environmental quality. When environment has a higher assimilating ability (ϵ is higher), the corresponding long-run environmental quality is higher as well. The concavity also reveals that the environmental quality will not explode as the ecological services provided in nature is limited (Smulders,

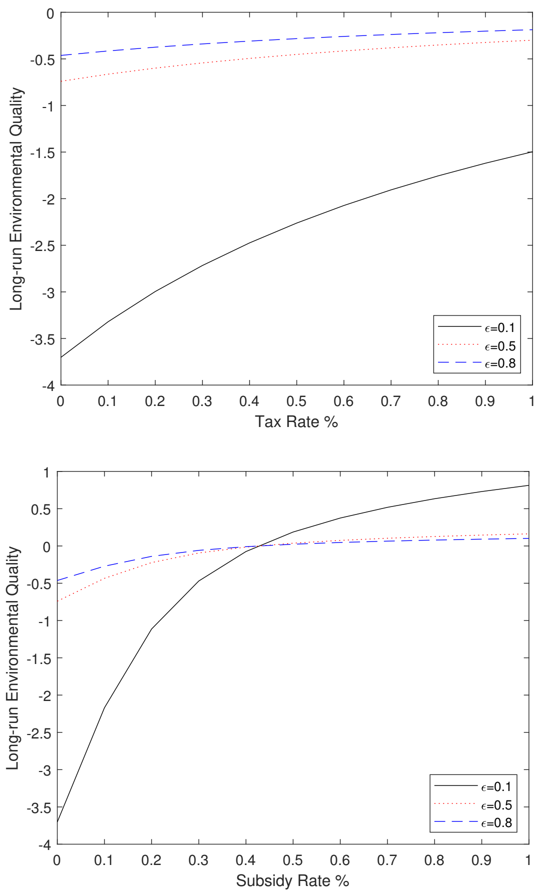

Figure 2.3: The effects of different policies on the environmental quality

2000).

Welfare

Household obtains utility from driving but also suffers from the pollution caused by vehicle driving. How much households value the environmental quality depends on several factors. We investigate whether the policies improve social welfare given different willingness to pay for the environmental quality. According to Jackson (1983), place of residence affects household's preference for the environmental quality: centre city has the benchmark preference value 1, suburb suffers more from pollution thus household living there puts more weight on the environmental quality, $\phi_3 = 1.4$. Rural areas have the least willingness to pay for better environmental quality, with $\phi_3 = 0.56$.

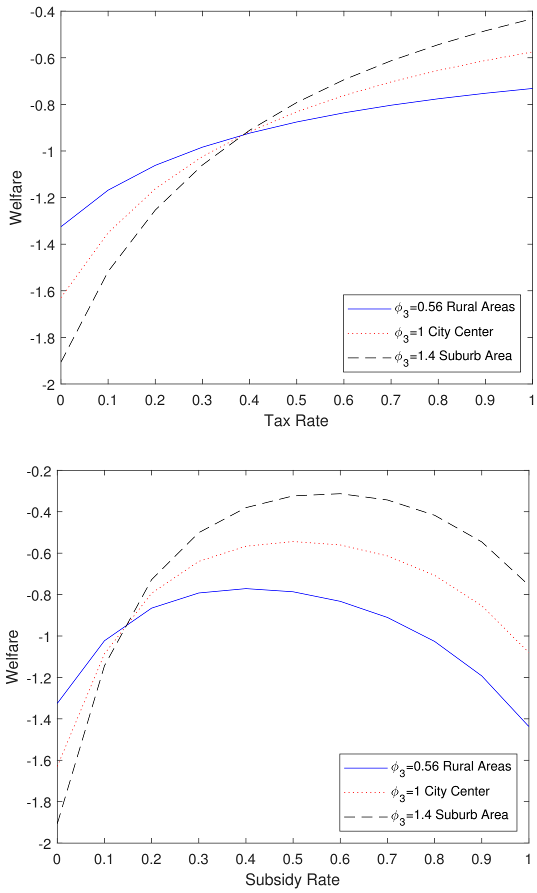

Figure 2.4: The effects of different policies on social welfare

Fig.2.4 shows the effects of fuel tax and subsidy rate on welfare. Three cases where people have different preferences for the environmental quality are measured in the figure. Overall, under fuel tax only policy and earmarking policy, the decreasing pollution level improves the environmental quality. Although higher fuel price makes consumers drive less, they switch their demand towards normal consumption good. Therefore, we observe the improvement of social welfare in the long run.

Long-run social welfare is more complex under the subsidy policy. Subsidies to fuel efficiency, on the one hand makes vehicles consume less fuel and consequently improves environmental quality, thereby mitigating the externality. On the other hand, subsidies cause a miss-allocation of resources on the production side (production inefficiency), which has a negative income effect (the economy produces inside its production possibilities frontier). In terms of welfare, the subsidies will improve welfare if the externality-mitigating effect dominates the production-inefficiency effect. Our numerical analysis suggests that for low levels of the subsidy the externality effect dominates, while for high levels of the subsidy production inefficiency becomes dominant. Consequently welfare is increasing in the subsidy for low levels and declining for high levels (inverted U shape).

2.6 Conclusion

This paper develops a new dynamic general equilibrium infinite-horizon model with vehicles which are of two vintages. Our aim is to analyse the endogenous determination of fuel consumption, fuel efficiency, households driving choices, environmental quality and social welfare under two policy options: 1) government levies tax on fuel and 2) government provides subsidy to firms to produce more fuel-efficient and cleaner engines. Our analyses demonstrate that at the steady state, households prefer to use new cars more often than old cars and fuel consumed by new cars is proportionally higher than fuel consumed by old cars.

Our numerical analysis show that providing subsidies to firms leads to more resources being allocated to the production of more fuel-efficient engines and less towards households' consumption and leisure which eventually decreases social welfare.

Instead, levying tax on fuel would not distort the production side of the economy. By increasing fuel tax rate, government can be assured to achieve fuel consumption reduction and pollution control. Environmental quality and social welfare in the long run improve.

2.7 Appendix

2.7.1 Optimality conditions derivation for household's problem

Household maximizes its discounted life time utility, as shown in Eq.2.2.22 subject to its budget constraint Eq.2.2.23.

Thus, the Lagrangian reads:

$$\mathcal{L} = \sum_{s=0}^{\infty} \beta^{t+s}[U(c_t, M_t, 1-l_t, N_t) + \lambda_{t+s}(\pi_t^a + \pi_t^g + w_t l_t + r_t k_t - (p_t + \tau_t)(g_{t,1} + g_{t,2}) \\ -k_{t+1} + (1-\epsilon_k)k_t - c_t - q_t^a(a_t \delta_t))]. \tag{2.A.1}$$

We then obtain the first-order conditions:

$$\frac{U_{c_t}}{U_{c_{t+1}}} = \beta(1-\epsilon_k + r_{t+1}), \tag{2.A.2}$$

$$\frac{\partial U_t}{\partial M_t}\frac{\partial M_t}{\partial m_{t,i}}\frac{\partial m_{t,i}}{\partial g_{t,i}} = U_{c_t}(p_t + \tau_t), \qquad i = 1, 2 \tag{2.A.3}$$

$$\beta\left(\frac{\partial U_{t+1}}{\partial M_{t+1}}\frac{\partial M_{t+1}}{\partial m_{t+1,1}}\frac{\partial m_{t+1,1}}{\partial a_t \delta_t}\right) + \beta^2\left(\frac{\partial U_{t+2}}{\partial M_{t+2}}\frac{\partial M_{t+2}}{\partial m_{t+2,2}}\frac{\partial m_{t+2,2}}{\partial a_t \delta_t}\right) = q_t^a U_{c_t}, \tag{2.A.4}$$

$$U_{1-l_t} = w_t U_{c_t}. \tag{2.A.5}$$

2.7.2 Proof for proposition 1

In the steady state, Eq.2.A.3 becomes:

$$\frac{\partial U}{\partial M} M_{ss}^{1-\sigma}\frac{m_{2,ss}^{\sigma}}{g_{2,ss}} = \frac{\partial U}{\partial M} M_{ss}^{1-\sigma}\frac{m_{1,ss}^{\sigma}}{g_{1,ss}}. \tag{2.7.6}$$

Eq.2.7.6 states that in the steady state, the marginal utility of fuel consumption for both types of vehicles are the same. Simplify Eq. 2.7.6 we can obtain Eq.2.3.25.

2.7.3 Proof for proposition 2

In the steady state, Eq.2.2.17 and 2.2.18 become:

$$m_{1,ss} = (a_{ss}\delta_{ss})^{\gamma} g_{1,ss}, \tag{2.7.7}$$

$$m_{2,ss} = (\rho a_{ss}\delta_{ss})^{\gamma} g_{2,ss}. \tag{2.7.8}$$

Thus, using Eq.2.3.25 we could get the mileage ratio between two types of vehicles in the equilibrium.

2.7.4 Calibration

Parameters related to driving service

The driving service is provided by two types of cars: old cars and new cars. ρ measures vehicle wear-out status which affects the fuel efficiency condition in the next period. We set ρ to be 0.9. σ measures the preferences over different types of cars and we set it to 0.5. According to Chen et al. (2006), consumption output ratio is around 0.65 and Ferdous et al. (2010) states that personal cars spending over household expenditure ratio is 6%. According to the steady state equilibrium of the economy, Eq.2.3.25 states the fuel consumption ratio between new cars and old cars. According to U.S. Energy Information Administration (2010) data, the newly produced light-duty vehicles fuel consumption is 921 gallon per vehicle in 2010 and 882 gallon per vehicle in 2009. Thus, we use the ratio of two years to proxy the fuel consumption ratio between two different types of cars. Using the ratio, we get:

$$\rho^{\frac{\gamma\sigma}{\sigma-1}} = 1.0452.$$

In the steady state, Eq.2.A.4 becomes:

$$\phi_1(1+\rho^{\frac{\gamma\sigma}{\sigma-1}})a\delta q^a = \beta\phi_2\gamma(\rho^{\frac{\gamma\sigma}{\sigma-1}}+\beta)g_2 c.$$

Using the consumption output ratio (0.65), personal cars spending over expenditure ratio (0.06) and substituting $\rho^{\frac{\gamma\sigma}{\sigma-1}} = 1.0452$ in, we obtain the value of $\gamma = 0.42$.

Parameters related to production technology

The second category of parameters relates to production sector. Parameter ε_k denotes the depreciation rate of physical capital and we set it to be 0.1. The mean value of the annual real fuel price p_t is 1.0872 dollar. The aggregate productivity level of both sector, A_1 and A_2 are normalized to 1. The parameter α is considered as the share of total income paid to owners of capital and we set α_1 and α_2 to 0.33 and 0.42 based on the calibration results from Wei (2013). μ is the marginal transformation of vehicle capital and fuel efficiency and we set that to 1 for simplicity.

Parameters related to the preference of households

The third category of parameter describes the preferences of household. The subjective discount rate β is 0.97. In the model, we assume log-preference for consumption, driving service, leisure and environmental quality shown in Eq.2.2.16, which implies that consumption and driving service are not perfect substitutes. We calibrate the parameters ϕ_1 and ϕ_2 to 0.34 and 0.05 respectively to match the fact that households allocate two thirds of their time in leisure.

Based on Ghez et al. (1975), households normally spend one third of their time to market activities(time not spent on sleeping or personal maintenance). The fuel consumption output ratio (Eq.2.A.3) in steady state becomes:

$$\phi_2 c = \phi_1 (p_t + \tau_t)(g_1 + g_2).$$

Dividing each side by output and using the consumption output ratio and fuel expenditure over income ratio, we calibrate the parameter ϕ_1 and ϕ_2 to 0.34 and 0.05.

Households benefit from good environmental quality. Jackson (1983) shows that although the household's willingness to pay for better environmental quality is sensitive to the model specification and other assumptions, the income elasticity is in the vicinity of 1. We thus set the benchmark value for ϕ_3 to 1.

Parameters related to environmental quality and pollution

The concept of environmental quality is depicted in Eq.2.2.20. Environmental quality is a stock which is improved every period by natural purification capacity and damaged by the pollution. ϵ measures nature's purifying capacity and we set it to 0.1. Pollution, as expressed in Eq.2.2.21, is positively related to fuel consumption but mitigated by the fuel efficiency condition δ. Parameter μ_1 and μ_2 measures to which extent the fuel efficiency help in addressing pollution caused by fuel consumption. For simplicity, we assume μ_1 and μ_2 to unity.

Chapter 3

Optimal Environmental Taxation in the Presence of Pollution and Congestion: A General Equilibrium Analysis

This chapter derives the optimal steady-state first-best environmental tax structure in the presence of (i) different vintage vehicles (new vehicles and old vehicles), (ii) pollution and congestion externalities caused by vehicle driving. Analytical results show that the optimal fuel tax is determined by two opposing forces caused by gasoline consumption: marginal cost of pollution and marginal cost of congestion. We also find that the optimal road taxes depend on the gasoline consumption ratio between the new cars and the old cars. We further derive the solution of a uniform fuel tax and examine how that affects the optimal road taxes accordingly. Our calibration on the U.S. economy shows that the optimal levels of transport taxes are affected by the households' preferences on the environmental factors. In the presence of congestion externality, optimal fuel tax for old vehicles is higher, which shows that the marginal cost of pollution outweighs the marginal cost of congestion when households start to value the environmental quality. When we implement uniform fuel tax, fuel tax rate lies between the optimal fuel taxes for new vehicles and old vehicles. We also find that long-run utility is higher under optimal fuel tax than uniform fuel tax but not to a substantial extent.

3.1 Introduction

Fuel consumed during driving creates externalities through air pollution, congestion, accidents and import dependence (Haughton and Sarkar, 1996). The guaranteeing of the efficiency of a competitive process and ways to address the externalities have been important issues for economic policy construction. Environmental taxes, internalizing the external costs which vehicle driving imposes on the rest of the society, have been a popular policy tool to address externalities (Bovenberg and De Mooij, 1994). However, what we observe is that fuels are taxed at widely different rates in different countries (Newbery, 2005), with the U.K. in particular standing out as having high oil taxes in contrast to the U.S. being specifically low in its oil taxes among all the OECD countries (OECD, 2018). This raises a curious question as to the appropriateness of the environmental taxes set by different countries.

This paper focuses on two important externalities generated by fuel via driving. The first external impact is pollution which is viewed as a byproduct of gasoline combustion during driving. The emissions of carbon dioxide, nitrogen oxides and monoxide pose great threats to residents especially in urban areas. The latter two are the main cause for smog while carbon dioxide accumulates and contributes to greenhouse effect which might contributes to global warming (Haughton and Sarkar, 1996). Poor air quality causes 40,000 to 50,000 early deaths in the U.K. and the cost of these health impacts is estimated at £20 billion every year. The World Health Organization (WHO) calculates that people in the U.K. are 64 times as likely to die of air pollution as those in Sweden and twice as likely as those in the U.S. [1]. The second externality caused by driving is congestion. Gasoline is mainly used in motor vehicles (Haughton and Sarkar, 1996) and the more often households drive their vehicles, the heavier the traffic. In the U.K., traffic congestion in largest cities is 14% worse than it was five years ago[2]. Many studies have shown that there is a strong link between air pollution and congestion caused by vehicle driving. Traffic congestion drastically worsens the air quality. In nose-to-tail traffic, tailpipe emissions are four times greater than they are in free flow traffic (Bell, 2006). During periods of heavy traffic, the falling speed of the traffic worsens air pollution. Morning peak traffic average speeds in central London have fallen from 16 kmph in 2006 to 12 kmph in 2016, causing a 10% increase in NO_x from diesel cars and vans, and a 25% and 27% increase for buses and trucks[3].

Governments have recognized the need to tackle traffic congestion and different policies have been put in practice[4]. Among all the tools, it is widely believed that

[1] World Health Statistics, Monitoring Health for the SDGs, World Health Organisation, 2017.

[2] Travel in London Report 9, Transport for London 2016.

[3] Travel in London Report 9, Transport for London 2016.

[4] Since 2008 Summer Olympics, Beijing started a license plate rationing scheme whereby each

economic incentives bring about a more efficient allocation of road space and natural resources (Walters, 1961).

A number of previous empirical studies have attempted different ways to quantify the external costs generated[5] and most of their estimation being mileage based. Parry and Small (2005) build up a static analytical framework and solve for tje second-best optimal fuel tax and decompose it into components that reflect different external costs. They then calibrate their model based on the U.K. and the U.S. economies to explain why different countries have different fuel tax rates. However, there are still limitations within the previous research. Firstly, congestion itself cannot be fully addressed by only taxing fuel. Congestion is normally measured by the time spent on the road. Driving time is determined as the inverse of average travel speed times mileage of travel (Parry and Small, 2005). As agents normally take average driving speed as a given, the higher the mileage of travel, the longer time agents have to spend on the road, hence heavier traffic. Mileage of travel is produced by different fuel-efficiency-level of cars and gasoline. Therefore, simply by charging higher price on fuel would not fully solve the congestion externality. Secondly, it is crucial to take into consideration of the endogeniety of fuel efficiency. As fuel becomes more expensive, households respond to this by either driving more fuel-efficient vehicles or driving less, which means that fuel economy of the vehicle

car is banned from urban core area one workday per week, depending on the last digit of its licence plate. In 2003, a congestion fee for driving in London was introduced and that year it was reported that the scheme resulted in a 10% reduction in traffic volumes and an overall reduction of 11% in vehicle kilometres in London.

[5] See Peirson et al. (1995), Mayeres et al. (1996), and Rothengatter and Mauch (2000).

fleet matter. Thirdly, fuel efficiency progresses over time and more fuel-efficient vehicles contributes less emissions thus lower pollution level. To capture the long-run impact of policy on environment, the contribution of a static model is very much diminished.

This paper contributes to the theoretical literature in several ways. First, to fully tackle the external cost generated by vehicle driving, we examine the first-best environmental taxes to address pollution and congestion externalities separately. We then examine the interrelation between optimal environmental taxes and how they affect optimal tax structure. Second, we introduce capital heterogeneity using "putty clay" technology (see Cooley et al. (1997) and Solow et al. (1960a)) to model vehicles of different vintage so as to better capture the impact of fuel efficiency endogeneity on optimal environmental taxes. Third, a dynamic view is useful in interpreting pollution externalities as emissions accumulate over time and impact agents in the long-run. This paper examines the first-best optimal environmental taxes (fuel taxes and road taxes) employing a two-period vintage dynamic general equilibrium model with pollution and congestion externalities presented.

We summarize the results as follows. First, analytical results show that the first-best optimal fuel taxes consist of two parts: marginal cost of pollution and marginal cost of congestion. New cars generate less pollution but contribute more to the mileage of travel which leads to more congestion. Thus, the optimal fuel taxes of different types of vehicles depend on these two contradicting factors. Optimal road taxes target at the congestion externality which is related to vehicle fuel efficiency level. In the steady states, households prefer to drive new cars more often which implies higher mileage of travel, thus road tax is higher for new cars than old cars. We further solve for uniform fuel tax and it takes the form of weighted average of fuel taxes of new cars and old cars. Second, we calibrate our model based on the U.S. economy and show that the optimal environmental taxes depend on the households' preferences on environmental factors. In the presence of congestion externality, optimal fuel tax for old vehicles is higher when households start to value environment which shows that the marginal cost of pollution outweighs the marginal cost of congestion. Households are better off under optimal fuel tax than uniform fuel tax but not to a substantial extent.

The paper is organized as follows. Section 3.2 describes the model. Section 3.4 solves the social planner's problem and describes its dynamics while section 3.3 looks at the decentralized economy case. Section 3.5 and 3.6 present the environmental taxes solutions (fuel taxes and road taxes). Section 3.7 describes calibration and numerically present the environmental taxes under different sets of preference parameters. Section 3.8 concludes.

3.2 Basic features of the model

In this section, we introduce how vehicle-related features are modelled before we present the social planner's problem and household's problem to solve for the optimal environmental taxes.

3.2.1 Driving behavior

Driving service is composed of mileage of travel produced by both new and old cars:

$$M_t = (m_{t,1}^{\sigma} + m_{t,2}^{\sigma})^{\frac{1}{\sigma}}, \tag{3.2.1}$$

where $0 < \sigma < 1$ and it measures the price elasticity of demand.

$m_{t,1}$ measures the mileage of travel of new cars and $m_{t,2}$ old cars at each time period. This setting follows Grossman and Helpman (1991) to guarantee that households prefer to drive both types of cars rather than just the new ones.

At each time period, the mileage of travel by the new cars $(a_{t-1}\delta_{t-1})$ and the old cars$(\rho a_{t-2}\delta_{t-2})$ [6] are:

$$m_{t,1} = (a_{t-1}\delta_{t-1})^{\gamma} g_{t,1}, \tag{3.2.2}$$

$$m_{t,2} = (\rho a_{t-2}\delta_{t-2})^{\gamma} g_{t,2}, \tag{3.2.3}$$

where $0 < \rho < 1$ and $0 < \gamma < 1$.

Mileage of travel has a linear relation to gasoline and depends on the efficiency condition of the engines which are measured by γ [7]. $1-\rho$ measures the depreciation rate for the vehicle after having been used for a period.

3.2.2 Transport externalities

In this chapter, we include environmental factors and specifically we focus on pollution and congestion caused by vehicle driving.

[6] The description of vehicles is further explained in later section.

[7] This formulation follows Wei (2013).

Congestion: Parry and Small (2005) use driving time as a measurement for congestion which is the product of the inverse of the average travel speed and miles of travel. They also assume that agents take the average travel speed fixed as they do not take account of their own impact on congestion. Following Parry and Small (2005), we use the sum of mileages[8] as a proxy for congestion externality.

$$N_t = m_{t,1} + m_{t,2}. \tag{3.2.4}$$

It captures the two sources of congestion: 1) the ownership of vehicles and 2) the driving service when provided with gasoline. Congestion enters into utility function for household as a negative externality (See Eq.3.3.21). In the utility function, $\bar{N}$ measures the road capacity which bears the negative impact from congestion. The figure below depicts the relation between road capacity and utility.

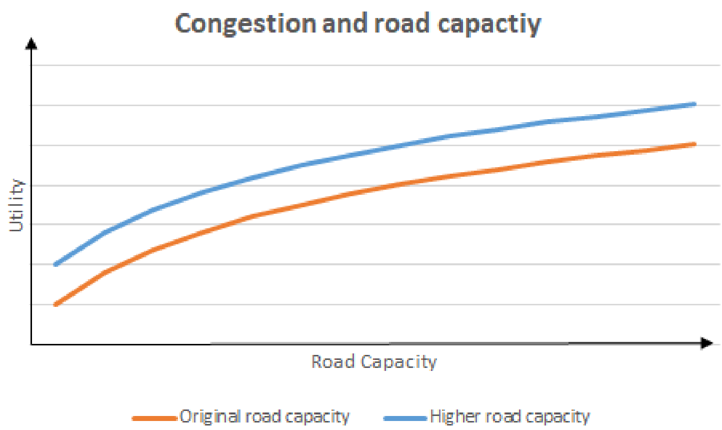

Figure 3.1: Congestion and road capacity

As seen from the figure, the x-axis measures road capacity$(\bar{N} - N)$ while y-axis measures the utility household gets from more spacious roads. When the roads

[8]Congestion is proportional to mileage and we abstract from peak hours. Congestion and pollution are not directly affecting each other but are related through the consumption of gasoline.

are less congested (lower N), households gains higher utility. Blue line depicts the scenario where a higher road capacity is realised (higher $\bar{N}$).

Pollution: Households gain utility from good environmental quality. However, gasoline combustion caused by vehicle driving leads to pollution which degenerates environmental quality. Meanwhile, at each period, nature assimilates a certain amount of pollutants and thus improves environment quality.

We base our formulation of pollution on Selden and Song (1995). The pollutant we focus here is local air pollutant which is caused by the usage of gasoline but mitigated by vehicle's embedded fuel-efficiency levels:

$$P_t = \frac{g_{t,1}}{\delta_{t-1}} + \frac{g_{t,2}}{\rho\delta_{t-2}}, \tag{3.2.5}$$

where $1 - \rho$ denotes the depreciation after vehicle being used for a period.

Eq.3.2.5 means that pollution is linear to gasoline consumed by both new cars and old cars, and new cars are more efficient in mitigating pollution as the marginal pollution caused by gasoline consumption is higher for old cars than new cars.

Environmental quality: Environmental quality is modelled as a type of asset. The quality of the environment, Q, represents the stock of natural capital and accumulates based on the regenerating ability of nature while depreciates due to pollution P. Q evolves over time according to the following function based on Bovenberg and Smulders (1995):

$$Q_{t+1} - Q_t = \Phi - \epsilon Q_t - P_t, \tag{3.2.6}$$

where $Q_{max} = \bar{Q}$ which implies that there is an upper limit for environmental quality. Φ represents the original level of environmental quality while ϵ denotes the nature's pollutant's assimilating rate. The law of motion shows that environmental quality is improved each period by nature's pollutants-assimilating ability.

3.3 Decentralized economy

Now we start to look at the scenario where we have many firms and many identical households. Households own all factors of production and all shares in firms. Government in the economy collects taxes from gasoline consumption and transfers the revenue back to households in a lump-sum payment.

3.3.1 Firms

There are two production sectors: one is the general production sector G which is used for general consumption goods c_t, accumulation of capital and the purchase of gasoline at an exogenous price p_t. The second is vehicle production sector F which produces vehicle capital a_t and fuel efficiency δ_t.

General production

In this sector, firms hire labours l_t^g and rent capital k_t^g from the households to produce consumption goods, accumulate capital and import gasoline at a fixed price with constant-return-to-scale technology. The profits generated go back to households. The resource constraint reads:

$$G(k_t^g, l_t^g) = c_t + k_{t+1} - (1 - \epsilon_k)k_t + p_t(g_{t,1} + g_{t,2}). \tag{3.3.7}$$

The problem facing the firms in this sector is to maximize its profit (π_t^g) with respect to capital k_t^g and labour l_t^g:

$$\max_{k_t^g, l_t^g} \pi_t^g = G(k_t^g, l_t^g) - r_t^g k_t^g - w_t^g l_t^g. \tag{3.3.8}$$

We normalize the price from the general production to unity. Given its constant-return-to-scale technology, the profit from the general production sector π_t^g will be zero.

The corresponding first-order conditions are:

$$r_t^g = G_{k_t^g}, \tag{3.3.9}$$

$$w_t^g = G_{l_t^g}. \tag{3.3.10}$$

Vehicle production

Vehicle is a capital good made up of two attributes, the chassis of the car a and the fuel efficiency component δ (the car engine power). Those two components are produced separately but must be sold as a combined product. Fuel efficiency δ, embedding clean technology, is crucial in mitigating air pollution (see Eq.3.2.5).

In this sector, firms hire labour l_t^a and rent capital k_t^a to produce vehicle capital a_t and fuel efficiency δ_t. The firms sell the combination of vehicle capital and fuel efficiency to households at price q_t^a. The resource constraint reads:

$$F(k_t^a, l_t^a) = a_t + \mu\delta_t, \tag{3.3.11}$$

where μ measures the marginal transformation rate between vehicle capital a_t and fuel efficiency δ_t.

Firm's goal is to maximize its profit (π_t^a) with respect to capital k_t^a, labour l_t^a

and how much fuel efficiency to produce δ_t:

$$\max_{k_t^a, l_t^a, \delta_t} \pi_t^a = q_t^a a_t \delta_t - r_t^a k_t^a - w_t^a l_t^a. \tag{3.3.12}$$

The first-order conditions for the vehicle producer are:

$$r_t^a = q_t^a \delta_t F_{k_t^a}, \tag{3.3.13}$$

$$w_t^a = q_t^a \delta_t F_{l_t^a}, \tag{3.3.14}$$

$$q_t^a [F(k_t^a, l_t^a) - \mu \delta_t] - \mu q_t^a \delta_t = 0. \tag{3.3.15}$$

Equilibrium conditions in the production sector

To clear the production sector:

$$k_t^a + k_t^g = k_t, \tag{3.3.16}$$

$$l_t^g + l_t^a = l_t, \tag{3.3.17}$$

where k_t represents the total capital and l_t the total labour at time period t.

$$w_t^a = w_t^g = w_t, \tag{3.3.18}$$

$$r_t^a = r_t^g = r_t. \tag{3.3.19}$$

3.3.2 Households

Representative household gains utility from general consumption c_t, driving service M_t, leisure $1 - l_t$ and environmental quality Q_t. They get disutility from congestion N_t. We assume that the utility function is concave and is twice continuousy differentiable:

$$U(c_t, M_t, 1 - l_t, N_t, Q_t). \tag{3.3.20}$$

We assume log-preferences for the utility function:

$$U(c_t, M_t, 1-l_t, N_t, Q_t) =$$
$$\phi_1 \log c_t + \phi_2 \log M_t + (1-\phi_1-\phi_2) \log(1-l_t) + \phi_3 \log(\bar{N} - N_t) + \phi_4 \log Q_t, \tag{3.3.21}$$

where ϕ_1, ϕ_2, ϕ_3 and ϕ_4 are all positive.

Each time period, the household supplies labour l_t and capital k_t to firms and receives all the profits generated by both types of firms (π_t^a and π_t^g). A lump-sum payment T_t is paid to the household from the government. The household spends on consumption goods, gasoline, new vehicles and investment. Household is also subject to environmental taxes: gasoline tax for new cars and old cars (τ_t^1 and τ_t^2) respectively and road taxes for new cars and old cars ($T_{t,1}$ and $T_{t,2}$).

Thus, the budget constraint facing households is:

$$\begin{aligned} \pi_t^a + \pi_t^g + w_t l_t + r_t k_t = (p_t + \tau_t^1) g_{t,1} + (p_t + \tau_t^2) g_{t,2} + k_{t+1} - (1-\epsilon_k) k_t + c_t \\ + q_t^a (a_t \delta_t) + T_{t,1}(a_{t-1}\delta_{t-1}) + T_{t,2}(a_{t-2}\delta_{t-2}) + T_t. \end{aligned} \tag{3.3.22}$$

The problem the household is facing is to maximize its discounted life-time utility:

$$\max_{c_t, g_{t,1}, g_{t,2}, a_t\delta_t, l_t} \sum_{s=0}^{\infty} \beta^{t+s} U(c_t, M_t, 1-l_t, N_t, Q_t), \tag{3.3.23}$$

subject to the budget constraint shown in Eq. 3.3.22.

Note that when making decisions, household does not internalize the detrimental effects caused by vehicle driving. Put differently, household does not consider externalities.

We solve the maximization problem by transforming it into the equivalent Bellman equation format:

$$V^t(k_t, a_{t-1}\delta_{t-1}, \rho a_{t-2}\delta_{t-2}; \{I_t, Q_t\}) =$$
$$\max_{c_t, g_{t,1}, g_{t,2}, a_t\delta_t, l_t} \Big[U(c_t, M_t, 1-l_t, N_t, Q_t) + \beta V^{t+1}(k_{t+1}, a_t\delta_t, \rho a_{t-1}\delta_{t-1}; \{I_{t+1}, Q_{t+1}\})\Big]. \tag{3.3.24}$$

The derivation of the first-order conditions and the envelope conditions are shown in Appendix 3.9.1.

3.3.3 Government

Government levies tax on household's purchase of gasoline and transfer the tax revenue back to households in a lump-sum payment.

$$T_t = \tau_t^1 g_{t,1} + \tau_t^2 g_{t,2} + T_{t,1}(a_{t-1}\delta_{t-1}) + T_{t,2}(a_{t-2}\delta_{t-2}). \tag{3.3.25}$$

3.4 Social planner's problem

We now move on to solve the social planner's problem where government allocates the resources. The Bellman equation to government's problem is:

$$V^t(k_t, Q_t; a_{t-1}\delta_{t-1}, \rho a_{t-2}\delta_{t-2}; \{I_t\}) =$$
$$\max_{c_t, g_{t,1}, g_{t,2}, a_t, l_t^g, l_t, k_t^g, k_t} \Big[U(c_t, M_t, 1-l_t, N_t, Q_t) + \beta V^{t+1}(k_{t+1}, Q_{t+1}; a_t\delta_t, \rho a_{t-1}\delta_{t-1}; \{I_{t+1}\})\Big], \tag{3.4.26}$$

subject to the resource constraints:

$$G(k_t^g, l_t^g) = c_t + k_{t+1} - (1-\epsilon_k)k_t + p_t(g_{t,1} + g_{t,2}),$$

$$F(k_t^a, l_t^a) = a_t + \mu\delta_t,$$

and environmental quality's law of motion:

$$Q_{t+1} - Q_t = \Phi - \epsilon Q_t - P_t.$$

The resource constraint (Eq.3.3.11) and the equilibrium condition (Eq.3.3.16) imply that:

$$\delta_t = H(k_t - k_t^g, l_t - l_t^g, a_t).$$

The optimality conditions are derived in Appendix 3.9.2.

3.5 Optimal environmental taxes

3.5.1 Optimal gasoline taxes

Taxes are used to correctly price social activities causing externalities, i.e. pollution and congestion. Gasoline taxes help prices closely approximate marginal social cost, that is, the gasoline tax household has to pay should equate exactly to the marginal social cost caused by gasoline consumption so as to achieve first best. Given that we have different types of vehicles, different and specific gasoline taxes need to be applied. Thus, using optimality conditions we obtained in both household's problem and social planner's problem, we are able to equalize the marginal social cost and the tax.

Eq.3.A.2 and Eq.3.A.11 render the optimal gasoline tax rate for new cars:

$$\tau_t^1 = \frac{V_{Q_{t+1}}^{t+1}}{V_{k_{t+1}}^{t+1}} \frac{\partial P_t}{\partial g_{t,1}} - \frac{U_{N_t} \frac{\partial N_t}{\partial m_{t,1}} \frac{\partial m_{t,1}}{\partial g_{t,1}}}{\beta V_{k_{t+1}}^{t+1}}. \tag{3.5.27}$$

Similarly, Eq.3.A.3 and Eq. 3.A.12 give us the optimal gasoline tax rate for old cars:

$$\tau_t^2 = \frac{V_{Q_{t+1}}^{t+1}}{V_{k_{t+1}}^{t+1}} \frac{\partial P_t}{\partial g_{t,2}} - \frac{U_{N_t} \frac{\partial N_t}{\partial m_{t,2}} \frac{\partial m_{t,2}}{\partial g_{t,2}}}{\beta V_{k_{t+1}}^{t+1}}. \tag{3.5.28}$$

The formulation of optimal gasoline taxes reveal that gasoline consumption contributes to both pollution and congestion. To see which rate is higher, in the steady state, $\tau^2 - \tau^1$ become:

$$\tau^2 - \tau^1 = \frac{\overset{+}{V_Q}}{V_k}\overset{+}{\left(\frac{1}{\rho\delta} - \frac{1}{\delta}\right)} + \frac{\overset{-}{U_N}}{\beta V_k}\overset{+}{[(a\delta)^\gamma - (\rho a\delta)^\gamma]}. \tag{3.5.29}$$

Thus, in the steady state, the magnitude of the gasoline tax is undetermined analytically. It depends on the opposing factors between marginal cost of pollution and marginal cost of congestion caused by gasoline consumption. New cars cause less pollution given the same amount of gasoline consumed but they do provide more mileage of travel which contribute more to congestion. It is clear thus when only environmental quality is considered, gasoline tax for old cars is higher than new ones. Similarly, when only congestion externality is considered, gasoline tax for new cars is higher than old cars. When both types of externalities are considered, the tax rate depends on the dominating factor.

3.5.2 Optimal road taxes

Road taxes are used to correct congestion externalities. Compare the value functions from social planner's problem and decentralized economy, we obtain:

$$U_{N_t} \frac{\partial N_t}{\partial m_{t,1}} \frac{\partial m_{t,1}}{\partial (a_{t-1}\delta_{t-1})} = -\beta V_{k_{t+1}}^{t+1} T_1, \tag{3.5.30}$$

and

$$U_{N_t}\frac{\partial N_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial(\rho a_{t-2}\delta_{t-2})} = -\beta V_{k_{t+1}}^{t+1}T_2. \tag{3.5.31}$$

The equations above then render the solutions to the optimal road taxes:

$$T_1 = -\frac{U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial(a_{t-1}\delta_{t-1})}}{\beta V_{k_{t+1}}^{t+1}}, \tag{3.5.32}$$

and

$$T_2 = -\frac{U_{N_t}\frac{\partial N_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial(\rho a_{t-2}\delta_{t-2})}}{\beta V_{k_{t+1}}^{t+1}}. \tag{3.5.33}$$

It is straightforward in the formulation that vehicles as a type of capital only contribute to congestion externality and the tax rate is exactly determined by the amount of marginal social damage. Similarly, in the steady state, we want to see the comparison between marginal congestion cost of new cars and old cars:

$$T_1 - T_2 = \frac{U_N}{\beta V_k}\gamma(a\delta)^{\gamma-1}(\rho^{\gamma-1}g_2 - g_1). \tag{3.5.34}$$

We can see that the result is undetermined and depends on the gasoline consumption ratio between new cars and old cars. When the gasoline consumption ratio between two types of vehicles $\frac{g_1}{g_2} > \rho^{\gamma-1}$, then the road tax for new cars should be higher than the road tax for old cars and vice versa.

3.6 Uniform gasoline tax

Levying different gasoline tax rates based on the type of vehicles is a difficult policy to implement[9]. Thus, we are interested in finding how environmental taxes change when gasoline tax is taxed uniformly across different types of vehicles.

[9] This reflects the practice of fuel taxes in many countries.

3.6.1 Optimal uniform gasoline tax

To have uniform gasoline tax, the left hand side of Eq.3.A.2 and Eq.3.A.3 must be forced to be the same. Thus, we have:

$$U_{M_t}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}} = U_{M_t}\frac{\partial M_t}{\partial m_{t,2}}\frac{m_{t,2}}{\partial g_{t,2}},$$

which gives us the condition on gasoline consumption ratio:

$$g_{t,1} = (\frac{a_{t-1}\delta_{t-1}}{\rho a_{t-2}\delta_{t-2}})^{\frac{\gamma\sigma}{1-\sigma}} g_{t,2} = (\frac{\rho a_{t-2}\delta_{t-2}}{a_{t-1}\delta_{t-1}})^{\frac{\gamma\sigma}{\sigma-1}} g_{t,2}. \tag{3.6.35}$$

We can express it in a general form:

$$g_{t,1} = \Phi(a_{t-1}\delta_{t-1}, \rho a_{t-2}\delta_{t-2}) g_{t,2}. \tag{3.6.36}$$

We now solve the social planner's problem by putting this ratio in as a new constraint which means that $g_{t,1}$ is not going to be a choice variable.

Notice that under the new constraint, both the change in $a_{t-1}\delta_{t-1}$ and $\rho a_{t-2}\delta_{t-2}$ affect the change in g_1.Thus, the envelope conditions Eq.3.A.19 and Eq.3.A.20 change as well.

$$\begin{aligned} V^t_{(a_{t-1}\delta_{t-1})} &= U_{M_t}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial(a_{t-1}\delta_{t-1})} + U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial(a_{t-1}\delta_{t-1})} + \rho\beta V^{t+1}_{(\rho a_{t-1}\delta_{t-1})} \\ &\left[U_{M_t}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}} + U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}} - \beta(p_t V^{t+1}_{k_{t+1}} + V^{t+1}_{Q_{t+1}}\frac{\partial P_t}{\partial g_{t,1}})\right]\frac{\partial g_{t,1}}{\partial(a_{t-1}\delta_{t-1})}, \end{aligned} \tag{3.6.37}$$

$$\begin{aligned} V^t_{(\rho a_{t-2}\delta_{t-2})} &= U_{M_t}\frac{\partial M_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial(\rho a_{t-2}\delta_{t-2})} + U_{N_t}\frac{\partial N_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial(\rho a_{t-2}\delta_{t-2})} \\ &\left[U_{M_t}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}} + U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}} - \beta(p_t V^{t+1}_{k_{t+1}} + V^{t+1}_{Q_{t+1}}\frac{\partial P_t}{\partial g_{t,1}})\right]\frac{\partial g_{t,1}}{\partial(\rho a_{t-2}\delta_{t-2})}, \end{aligned} \tag{3.6.38}$$

while all the other first-order conditions remain the same (See Appendix 3.9.3 for derivation of constrained social planner's problem.).

Now we can solve the uniform gasoline tax. For household in the decentralized economy, they still make decisions separately on the consumption of gasoline (g_1 and g_2). However, they are now facing a uniform tax τ_t on gasoline in stead of separate ones.

Thus, Eq.3.A.2 and Eq.3.A.3 change into:

$$U_{M_t}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}} = \beta V_{k_{t+1}}^{t+1}(p_t+\tau_t), \tag{3.6.39}$$

$$U_{M_t}\frac{\partial M_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial g_{t,2}} = \beta V_{k_{t+1}}^{t+1}(p_t+\tau_t). \tag{3.6.40}$$

Substitute these into Eq. 3.A.27, we get:

$$\beta V_{k_{t+1}}^{t+1}(p_t+\tau_t)g_{t,2} + U_{N_t}\frac{\partial N_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial g_{t,2}}g_{t,2} - \beta\left(p_t V_{k_{t+1}}^{t+1} + V_{Q_{t+1}}^{t+1}\frac{\partial P_t}{\partial g_{t,2}}\right)g_{t,2}$$
$$+ \beta V_{k_{t+1}}^{t+1}(p_t+\tau_t)g_{t,1} + U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}}g_{t,1} - \beta\left(p_t V_{k_{t+1}}^{t+1} + V_{Q_{t+1}}^{t+1}\frac{\partial P_t}{\partial g_{t,1}}\right)g_{t,1} = 0,$$

which renders the solution for uniform tax rate:

$$\tau_t = \frac{g_{t,2}}{g_{t,1}+g_{t,2}}\tau_t^2 + \frac{g_{t,1}}{g_{t,1}+g_{t,2}}\tau_t^1. \tag{3.6.41}$$

The uniform gasoline tax rate takes the form of a weighted average of the gasoline tax rates for new cars and old cars.

3.6.2 Optimal road taxes under the constraint

Given that the new gasoline consumption ratio is in place, road taxes for new cars and old cars change accordingly as well. We match Eq.3.A.8 with Eq.3.6.37, and

Eq.3.A.9 with Eq.3.6.38 to get the adjusted road use taxes:

$$U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial(a_{t-1}\delta_{t-1})}+\left[\beta V^{t+1}_{k_{t+1}}\tau_t+U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}}-\beta V^{t+1}_{Q_{t+1}}\frac{\partial P_t}{\partial g_{t,1}}\right]\frac{\partial g_{t,1}}{\partial(a_{t-1}\delta_{t-1})}$$
$$=-\beta V^{t+1}_{k_{t+1}}T^c_1. \quad (3.6.42)$$

Similarly, match Eq.3.A.20 and Eq.3.A.9, we get:

$$U_{N_t}\frac{\partial N_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial(\rho a_{t-2}\delta_{t-2})}+\left[\beta V^{t+1}_{k_{t+1}}\tau_t+U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}}-\beta V^{t+1}_{Q_{t+1}}\frac{\partial P_t}{\partial g_{t,1}}\right]\frac{\partial g_{t,1}}{\partial(\rho a_{t-2}\delta_{t-2})}$$
$$=-\beta V^{t+1}_{k_{t+1}}T^c_2. \quad (3.6.43)$$

T^c_1 and T^c_2 denote road use taxes under constrained condition for new cars and old cars.

Eq.3.5.27 implies that:

$$\beta V^{t+1}_{k_{t+1}}\tau_t=\beta V^{t+1}_{Q_{t+1}}\frac{\partial P_t}{\partial g_{t,1}}-U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}}.$$

Substitute the above equation into the two expressions above, we get:

$$U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial(a_{t-1}\delta_{t-1})}+\beta V^{t+1}_{k_{t+1}}(\tau_t-\tau^1_t)\frac{\partial g_{t,1}}{\partial(a_{t-1}\delta_{t-1})}=-\beta V^{t+1}_{k_{t+1}}T_1,$$
$$U_{N_t}\frac{\partial N_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial(\rho a_{t-2}\delta_{t-2})}+\beta V^{t+1}_{k_{t+1}}(\tau_t-\tau^1_t)\frac{\partial g_{t,1}}{\partial(\rho a_{t-2}\delta_{t-2})}=-\beta V^{t+1}_{k_{t+1}}T_2.$$

Dividing $-\beta V^{t+1}_{k_{t+1}}$ on both sides of the above equations, we get the formulas for the road taxes under the gasoline consumption ratio constraint:

$$T^c_1=-\frac{U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial(a_{t-1}\delta_{t-1})}}{\beta V^{t+1}_{k_{t+1}}}-(\tau_t-\tau^1_t)\frac{\partial g_{t,1}}{\partial(a_{t-1}\delta_{t-1})}, \quad (3.6.44)$$

$$T^c_2=-\frac{U_{N_t}\frac{\partial N_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial(\rho a_{t-2}\delta_{t-2})}}{\beta V^{t+1}_{k_{t+1}}}-(\tau_t-\tau^1_t)\frac{\partial g_{t,1}}{\partial(\rho a_{t-2}\delta_{t-2})}. \quad (3.6.45)$$

With Eq. 3.5.32 and Eq.3.5.33, we can rearrange the expressions for the optimal road taxes into:

$$T^c_1=T_1-(\tau_t-\tau^1_t)\frac{\partial g_{t,1}}{\partial(a_{t-1}\delta_{t-1})}, \quad (3.6.46)$$

$$T_2^c = T_2 - (\tau_t - \tau_t^1)\frac{\partial g_{t,1}}{\partial(\rho a_{t-2}\delta_{t-2})}. \tag{3.6.47}$$

Therefore, the road taxes under uniform gasoline tax contain two parts: the original expression of road tax and the extra term which measures the marginal change made to gasoline consumption when vehicle types change. We are interested to know whether the new road taxes level are above or below the original level.

Given Eq.3.6.41, Eq.3.5.27 and Eq.3.5.28, we obtain $\tau^2 - \tau^1$ in the steady state:

$$\tau^2 - \tau^1 = \frac{\overset{+}{V_Q}}{V_k}(\frac{1}{\rho\delta} \overset{+}{-} \frac{1}{\delta}) + \frac{\overset{-}{U_N}}{\beta V_k}[(a\delta)^\gamma \overset{+}{-} (\rho a\delta)^\gamma]. \tag{3.6.48}$$

The sign of $\tau - \tau^1$ can not be determined. Thus, it still remains unknown analytically whether the road taxes under new constraint are above or below the ones without. To have a better picture of the tax rates and their interactions among each other under the gasoline consumption constraint, numerical simulation is needed.

3.7 Numerical solutions to the optimal environmental taxes

In this section, we employ a numerical model based on the U.S. economy to examine the first-best optimal environmental taxes. Calibrated model helps to relax the restrictions of the analytical model and assesses the economy in a more realistic setting. The calibration mostly follows the benchmark calibration we did in the first chapter with only a few changes.

3.7.1 Calibration

The table below summarizes the values of parameter in the calibration. The main change happens in household preference and environmental factor.

Category	**Parameters Description**	**Notation**	Value
Driving Service	Vehicle leftover rate	ρ	0.9
	Vehicle preference	σ	0.5
	Mileage production technology	γ	0.5
Production Technology	Capital depreciation rate	ϵ_k	0.1
	Capital share in production	α_1,α_2	0.33/0.42
	Productivity level	A_1, A_2	1
	Marginal transformation rate	μ	1
	Gasoline price	p_t	1.0872
Household Preference	Subjective discount rate	β	0.97
	Weight on consumption	ϕ_1	0.34
	Weight on driving	ϕ_2	0.05
	Weight on environmental quality	ϕ_4	1
	Marginal cost of congestion	ϕ_3	0.0127
Environmental Factor	Natural purifying capacity	ϵ	0.01
	Initial stock of environmental quality	Φ	10
	Congestion Extreme	$\bar{N}$	1

Table 3.1: Calibration

Household Preference

We assumed log-preference for the household as shown in Eq. 3.3.21:

$$U(c_t, M_t, 1-l_t, N_t, Q_t) =$$
$$\phi_1 \log c_t + \phi_2 \log M_t + (1-\phi_1-\phi_2)\log(1-l_t) + \phi_3 \log(\bar{N}-N_t) + \phi_4 \log Q_t.$$

At each time period, household gains utility from consumption c_t, driving M_t and leisure $1-l_t$. Household also benefits from environmental quality Q_t and suffers from congestion N_t. Parameter ϕ_1 and ϕ_2 are calibrated to 0.34 and 0.05 following Wei (2013) to match the fraction of time spent on market activities. How households value environmental quality is mostly geographically determined. We set the benchmark value to 1 to match with the city center scenario (Jackson, 1983).

Congestion arises because additional vehicles reduce the speed of other vehicles, and hence increase households' driving time. The average driving speed is a constant given that the road condition is fairly good. Therefore, an increase in aggregate vehicle miles of travel implies more congestion. The marginal cost of congestion to household is measured by ϕ_3. Based on Newbery (1990), we calibrate the congestion cost to be 0.0127 [10].

Environmental factor

Environmental quality, as shown in Eq.3.2.6, is a stock variable which changes over time based on the pollution caused by vehicle driving. ϵ measures the natural

[10] The formula for estimating marginal congestion cost comes from Department of Transport(US).

pollutant-absorbing ability and we set it to 0.01. Φ denotes the beginning level of environmental quality and we set it to 10.

3.7.2 Optimal environmental taxes

As shown in Eq.3.5.27 and Eq.3.5.28, gasoline is involved in generating both type of externalities. Old cars should be taxed more for generating more pollution while new cars should be taxed more for causing more traffic. We start from the benchmark calibration where household do not get affected by externalities ($\phi_3 = 0$, $\phi_4 = 0$). We then change the preference value of congestion (ϕ_3) and environmental quality (ϕ_4) to see its impact on optimal tax rates.

Table 3.2 shows the benchmark scenario where households do not take externalities into consideration (thus the preferences for congestion ϕ_3 and environmental quality ϕ_4 are zero). The optimal gasoline taxes (τ^1 and τ^2) and optimal road taxes (T_1 and T_2) are all zero. Households use new cars more often and therefore new cars provide higher mileage of travel ($m_1 > m_2$). New cars consume more gasoline than old ones ($g_1 > g_2$). Households do not pay for road use taxes and only pay for gasoline at its original price p_t. Next, we are going to include pollution externality into household's preference to see how the economy is going to change from the benchmark scenario.

Table 3.3 shows the economy when household cares only about pollution and thus congestion is excluded ($\phi_3 = 0$, $\phi_4 = 0.34$). Road taxes for both types of vehicles are still zero as congestion does not concern household. As only pollution externality is considered and new cars have a higher pollution mitigating ability

Benchmark: no externality

Variable	Description	Value	Variable	Description	Value
Economy in the Steady State					
c	consumption	0.3676	a	vehicle capital	0.1508
g_1	gasoline (new cars)	0.0255	δ	vehicle efficiency	0.1508
g_2	gasoline (old cars)	0.0242	l^a	labour (vehicle production)	0.0039
k^g	capital (general production)	1.4770	l^g	labour(general production)	0.3716
k^a	capital (vehicle production)	0.1657	l	total labour	0.3755
k	total capital	1.6427	P	pollution	0.3474
Optimal Environmental Taxation					
τ^1	optimal fuel tax (new cars)	0	τ^2	optimal fuel tax (old cars)	0
T_1	optimal road tax (new cars)	0	T_2	optimal road tax (old cars)	0
Mileage of Travel					
m_1	mileage travel by new cars			0.0039	
m_2	mileage travel by old cars			0.0035	
Travel Cost					
$(p_t + \tau^1)g_1$	gasoline cost for new cars			0.0277	
$(p_t + \tau^2)g_2$	gasoline cost for old cars			0.0263	
$T_1(a\delta)$	road tax cost for new cars			0	
$T_2(a\delta)$	road tax cost for old cars			0	
q^a	vehicle price			1.1353	
$q^a a\delta$	vehicle purchase cost			0.0258	

Table 3.2: Optimal environmental taxes and economy (benchmark calibration)

than old cars, Eq.3.5.27 and Eq.3.5.28 indicate that the optimal gasoline tax rate should be higher for old cars. This is further demonstrated by numerical results as the gasoline tax for new cars is \$0.1205/gallon and \$0.1339/gallon for old cars. Compared to the benchmark scenario where households ignore both externalities

Pollution externality only					
Economy in the steady state					
Variable	**Description**	**Value**	**Variable**	**Description**	**Value**
c	consumption	0.3694	a	vehicle capital	0.1512
g_1	gasoline (new cars)	0.0232	δ	vehicle efficiency	0.1512
g_2	gasoline (old cars)	0.0215	l_a	labour (vehicle production)	0.0039
k_g	capital (general production)	1.4649	l_g	labour (general production)	0.3686
k_a	capital (vehicle production)	0.1665	l	total labour	0.3725
k	total capital	1.6315	P	pollution	0.3117
Optimal Environmental Taxation					
τ^1	optimal fuel tax (new cars)	0.1205	τ^2	optimal fuel tax (old cars)	0.1339
T_1	optimal road tax (new cars)	0	T_2	optimal road tax (old cars)	0
Mileage of Travel					
m_1	mileage travel by new cars			0.0035	
m_2	mileage travel by old cars			0.0031	
Travel Cost					
$(p_t + \tau^1)g_1$	gasoline cost for new cars			0.0280	
$(p_t + \tau^2)g_2$	gasoline cost for old cars			0.0263	
$T_1(a\delta)$	road tax cost for new cars			0	
$T_2(a\delta)$	road tax cost for old cars			0	
q^a	vehicle price			1.1353	
$q^a a\delta$	vehicle purchase cost			0.0260	

Table 3.3: Optimal environmental taxes and economy: pollution externality only

(see Table 3.2), gasoline consumption decreases for both types of vehicles but to a different extent. New cars' gasoline consumption decreases by 9% while old cars' gasoline consumption decreases by 11%. However, new cars still consume more gasoline and new cars provide more mileage of travel than old ones. Fuel efficiency and vehicle capital increases and pollution decreases. Households still prefer to use new cars than old ones.

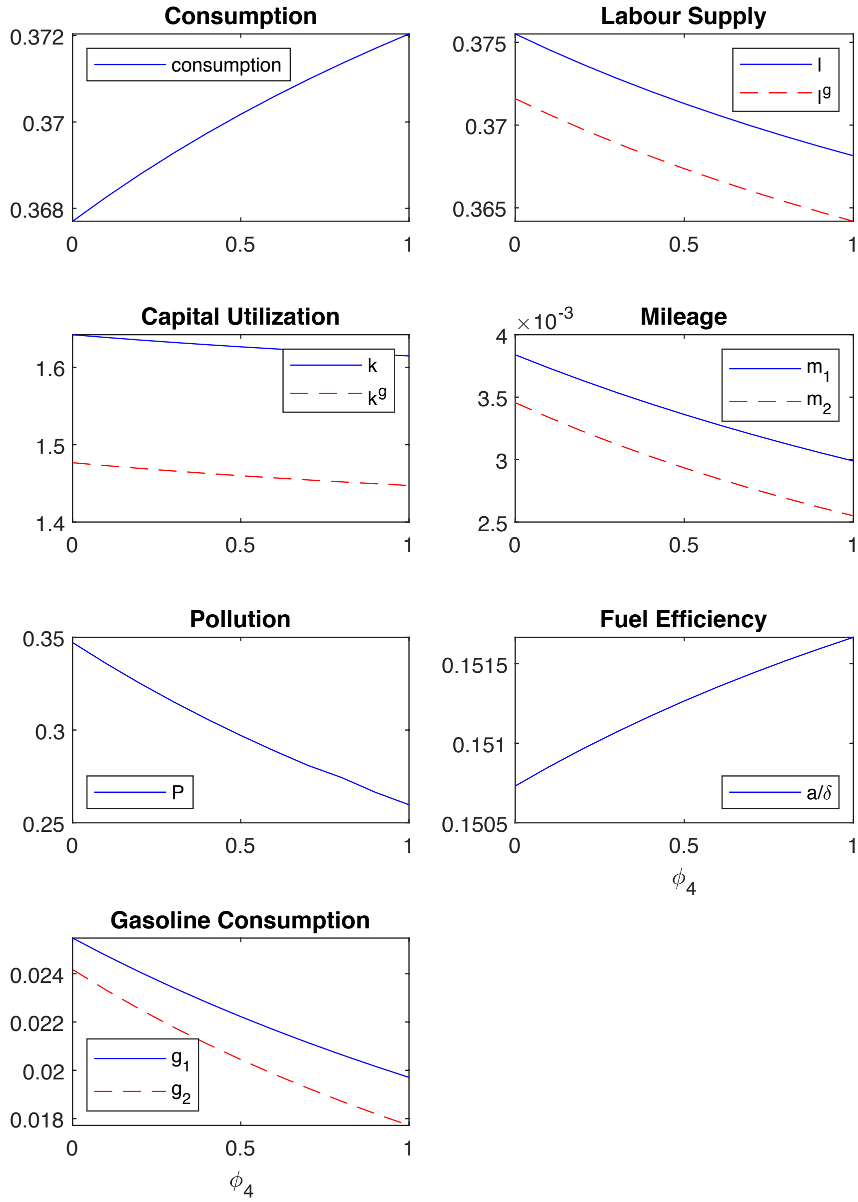

Figure 3.2: Optimal fuel taxes: the economy when preference for environment varies

We then examine the scenario where both externalities are concerned but household's preference for environmental quality (ϕ_4) varies. Figure 3.2 shows how economic variables are endogenously affected in the long run given different preferences on the environmental quality, ranging from zero to one. As the weight given to the environment increases, the marginal benefit gained from improving environment increases which induces consumers to switch their demand from driving towards consumption and leisure, which explains the positive increment in consumption and decline in labour supply. Gasoline consumption thus decreases for both type of vehicle and the drop for old cars is slightly bigger than for the new ones. Pollution decreases as a result of the decreasing gasoline consumption and vehicle service usage.

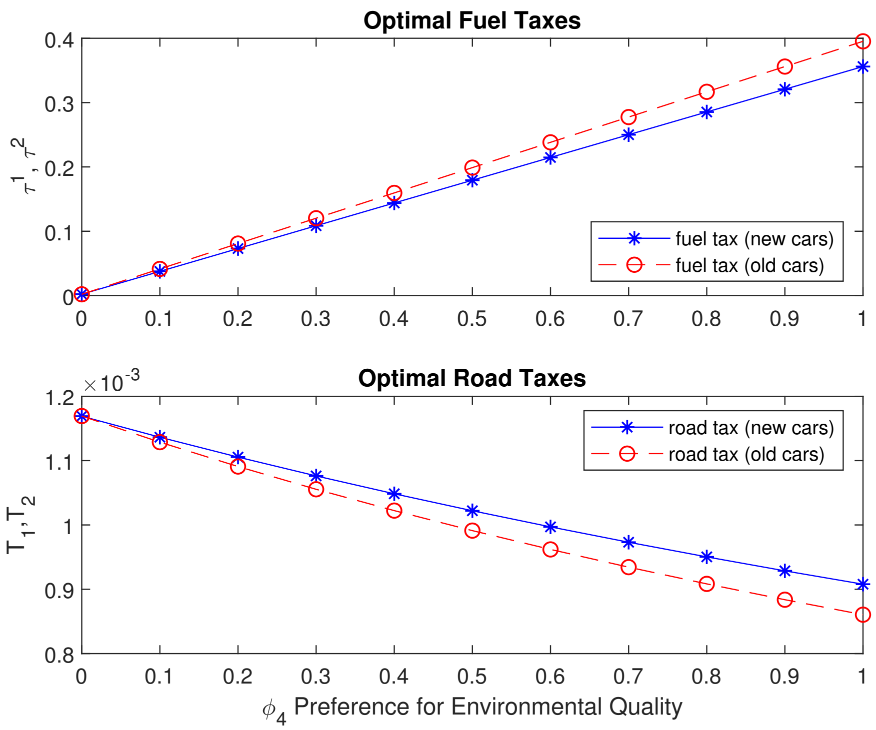

Figure 3.3: Optimal environmental taxation with different preference for environmental quality

Figure 3.3 shows the corresponding optimal fuel taxes and road taxes when ϕ_4 varies. Fuel taxes, as we discussed before in the analytical solution (Eq.3.5.27 and

Eq.3.5.28), depend on two contradicting factors: marginal cost of pollution and marginal cost of congestion caused by fuel consumption. New cars are more environmentally friendly but also contribute more to the traffic for being more efficient in providing mileage of travel. Numerical simulation suggests that, when both externalities are considered, the pollution mitigation ability dominates the congestion cost as household starts to care more and more about the environment. Old cars, therefore, are facing higher fuel tax than new cars. Optimal road taxes, as discussed before (Eq.3.5.32 and Eq.3.5.33), depend on the gasoline consumption ratio. As household cares more about environment, gasoline consumed by old cars decreases faster which makes gasoline consumption ratio increase. New cars provide more mileage of travel to household which implies more congestion. Therefore, road tax is higher for new cars than old cars.

3.7.3 Uniform gasoline tax

Levying different gasoline taxes based on vehicle type is difficult to implement in practice. We therefore solve for uniform gasoline tax under the fuel consumption ratio between new cars and old cars (Eq.3.6.41). We obtain the solution to social planner's problem under the constraint of gasoline consumption ratio (Eq.3.6.35). in the steady state, the constraint reduces to:

$$g_1 = \rho^{\frac{\gamma\sigma}{\sigma-1}} g_2. \tag{3.7.49}$$

Under bench mark calibration, the gasoline ratio $\frac{g_1}{g_2} = \rho^{\frac{\gamma\sigma}{\sigma-1}} > 1$, which means that new cars consume more gasoline than old cars. We then focus on the scenario where household takes both externalities into consideration, solve for the uniform

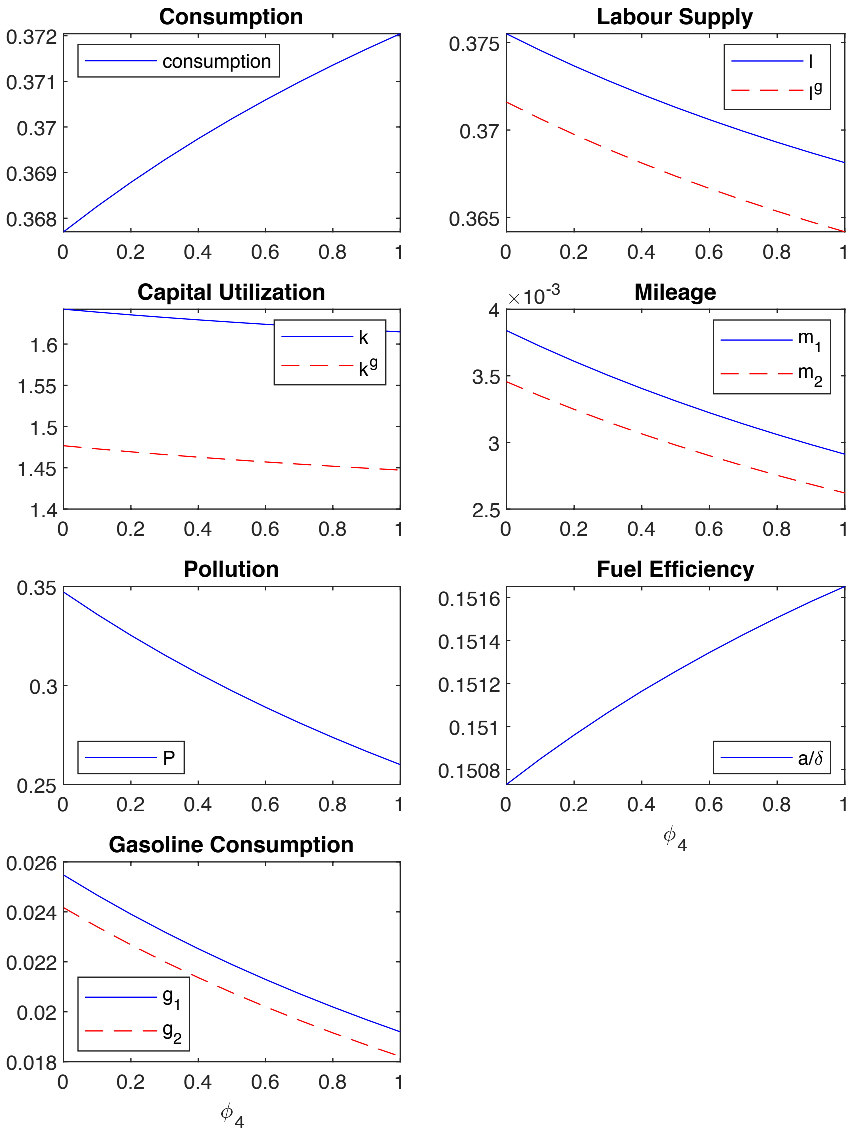

Figure 3.4: Uniform fuel tax: the economy when preference for environment varies

fuel tax and compare it to the optimal fuel taxes in the previous case. Table 3.A.2 describes the economy in the steady state in the presence of both externalities with varying preferences for environment quality. The gasoline consumption ratio constraint is very close to the optimal tax scenario, thus when the preference for the environment varies, the change of the economic variables in the steady state follows the same pattern (See Figure 3.4). As people care more about the environment, the increasing marginal damage from pollution makes household switch their demand towards consumption and leisure. We do observe that instead of gasoline consumed by old cars decreasing more, the gasoline consumption ratio keeps constant as shown in Eq.3.7.49.

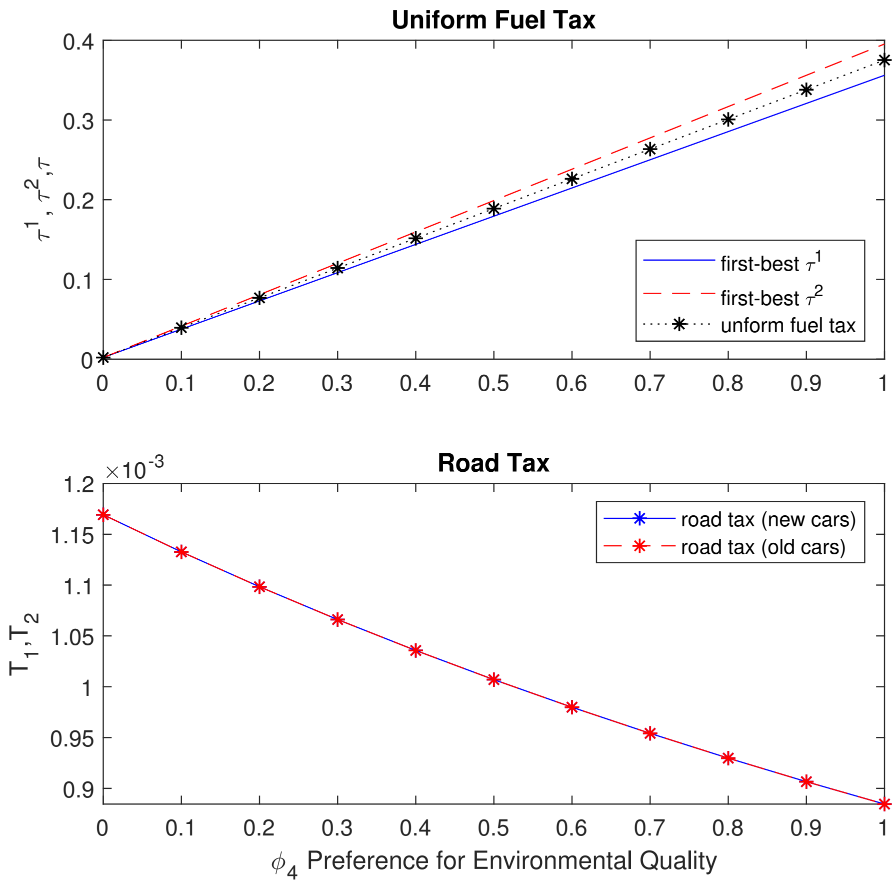

Figure 3.5: Uniform fuel tax and the corresponding road taxes with different preference for enviromental quality

The uniform fuel tax and corresponding road use taxes are shown in Figure 3.5. As shown in Eq.3.6.41, uniform fuel tax takes the form of a weighted average of fuel tax for new cars and old cars and thus uniform tax lies in between the two first-best fuel taxes. Figure 3.5 shows that as preference for environment grows, uniform gasoline tax increases and lies between fuel tax for new cars and old cars. Road taxes, based on calibration, are the same for both new cars and old cars and keep decreasing when households value the environment more.

3.7.4 Welfare

In this section, we compare the welfare status under optimal gasoline taxes and uniform gasoline tax. The difference between the optimal gasoline taxes for different types of vehicles and uniform gasoline tax is that we impose the gasoline consumption ratio. Given that the gasoline ratio constraint is quite close to what we have in the optimal fuel tax scenario, we do not observe huge differences in economy in the long-run, which means that the welfare does not vary too much.

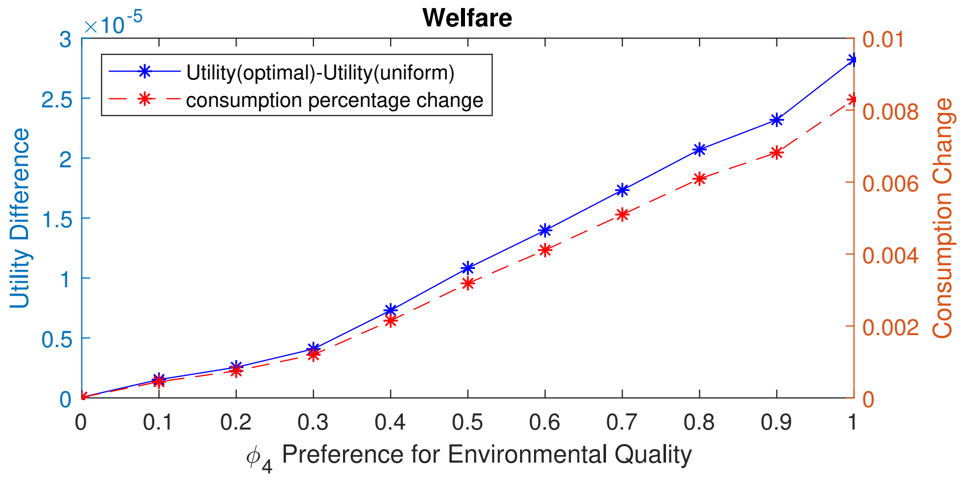

Figure 3.6: Utility difference and consumption change under optimal fuel taxes and uniform fuel tax

As shown in Figure 3.6, households are better off under optimal gasoline taxes but not to a large extent. As preference for environmental quality increases, the utility difference gap between the two policy options widens. We also solve for the consumption equivalence for one percentage improvement in environmental quality. Households gain utility from consumption c, driving service M, leisure $1-l$ and environmental quality Q while they suffer from congestion externality N. The percentage change of consumption $\frac{dc}{c}$ is expressed as:

$$\frac{dc}{c} = -\frac{\phi_4}{\phi_1}\frac{dQ}{Q}. \tag{3.7.50}$$

The expression depends on the preference ratio between consumption and environmental quality. We look at the equivalent percentage change of consumption when environmental quality changes:

Consumption percentage change

	$\phi_4 = 0.1$	$\phi_4 = 0.34$	$\phi_4 = 1$
dQ/Q	6.2085e-06	1.7547e-05	3.08e-05
dc/c	-1.8260e-06	-1.7547e-05	-9.0591e-05

Table 3.4: Consumption equivalence when environmental quality improves

Table 3.4 describes how much consumption household is willing to sacrifice in order to have environmental quality improved. As households value environment more, they are willing to sacrifice more consumption.

3.8 Conclusion

This paper employs analytical and numerical models to examine the general equilibrium interactions between gasoline taxes and road taxes to account for the externalities caused by vehicle driving: pollution and congestion.

The analytical model extends earlier work in many aspects. First, earlier work mainly focuses on gasoline tax and tries to only use gasoline tax to address all the externalities caused by gasoline consumption while neglecting the interaction among different environmental taxes. The model we built focuses on the externalities (pollution and congestion) caused by vehicle driving and we introduce different environmental taxes targeting at specific externalities. Second, we look into how tax rates differ when it comes to vehicles of different vintages.

This model indicates that in the presence of pollution and congestion, optimal fuel taxes depend on two contradicting powers: the marginal cost of pollution and marginal cost of congestion. New cars are more efficient in mitigating pollution but contribute more to traffic given its efficiency in providing mileage of travel. Old cars emit more pollutants but people are less willing to use them. Optimal road taxes depend on gasoline consumption ratio. We also solve for the environmental taxes when uniform fuel tax is implemented. Our model suggests that the uniform fuel tax is a weighted average of the first-best fuel taxes. Road taxes are undetermined due to the gasoline consumption ratio imposed.

The numerical simulations based on the U.S. economy support the analytical result. When households are concerned with both pollution and congestion, the marginal cost of pollution outweighs the marginal cost of congestion as households value environment more and more which leads to the optimal fuel tax for old cars being higher than for new ones. Under central values for parameters, gasoline consumption ratio between new cars and old cars is larger than one which indicates that road tax for new cars is higher. When implementing uniform fuel tax, we find that the gasoline consumption ratio is very close to the first-best case which indicates that the change of economic status is quite small. Numerical analysis shows that the uniform fuel tax is the weighted average of first-best fuel taxes and road taxes are the same for both new cars and old cars under the benchmark calibration.

In addition, we also analyse the welfare level under both optimal fuel taxes and uniform fuel tax. Given that the gasoline consumption restraint is very close to first-best scenario, household is only slightly better off in optimal fuel taxes case and the consumption equivalence change is also quite small. These considerations suggest that estimates of optimal fuel tax should also take other environmental taxes into consideration as they are intrinsically interdependent.

3.9 Appendix

3.9.1 Optimality conditions derivation to household's problem

The first-order conditions are:

$$c_t: \quad U_c = \beta V^{t+1}_{k_{t+1}}, \tag{3.A.1}$$

$$g_{t,1}: \quad U_{M_t} \frac{\partial M_t}{\partial m_{t,1}} \frac{\partial m_{t,1}}{\partial g_{t,1}} = \beta V^{t+1}_{k_{t+1}} (p_t + \tau^1_t), \tag{3.A.2}$$

$$g_{t,2}: \quad U_{M_t} \frac{\partial M_t}{\partial m_{t,2}} \frac{\partial m_{t,2}}{\partial g_{t,2}} = \beta V^{t+1}_{k_{t+1}} (p_t + \tau^2_t), \tag{3.A.3}$$

$$a_t \delta_t: \quad q^a_t V^{t+1}_{k_{t+1}} = V^{t+1}_{a_t \delta_t}, \tag{3.A.4}$$

$$l_t: \quad U_{1-l_t} = -w_t \beta V^{t+1}_{k_{t+1}}. \tag{3.A.5}$$

Similarly, we could get the envelope conditions:

$$V^t_{k_t} = \beta(1 - \epsilon_k + r_t) V^{t+1}_{k_{t+1}}, \tag{3.A.6}$$

$$V_{Q_t} = U_{Q_t} + \beta(1+\epsilon) V^{t+1}_{Q_{t+1}}, \tag{3.A.7}$$

$$V^t_{(a_{t-1}\delta_{t-1})} = U_{M_t} \frac{\partial M_t}{\partial m_{t,1}} \frac{\partial m_{t,1}}{\partial (a_{t-1}\delta_{t-1})} + \rho\beta V^{t+1}_{(\rho a_{t-1}\delta_{t-1})} - \beta V^{t+1}_{k_{t+1}} T_1, \tag{3.A.8}$$

$$V^t_{(\rho a_{t-2}\delta_{t-2})} = U_{M_t} \frac{\partial M_t}{\partial m_{t,2}} \frac{\partial m_{t,2}}{\partial (\rho a_{t-2}\delta_{t-2})} - \beta V^{t+1}_{k_{t+1}} T_2. \tag{3.A.9}$$

3.9.2 Optimality conditions derivation to government's problem

The first-order conditions read:

$$c_t: \quad U_c = \beta V_{k_{t+1}}^{t+1}, \tag{3.A.10}$$

$$g_{t,1}: \quad U_{M_t}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}} + U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}} = \beta\left(p_t V_{k_{t+1}}^{t+1} + V_{Q_{t+1}}^{t+1}\frac{\partial P_t}{\partial g_{t,1}}\right), \tag{3.A.11}$$

$$g_{t,2}: \quad U_{M_t}\frac{\partial M_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial g_{t,2}} + U_{N_t}\frac{\partial N_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial g_{t,2}} = \beta\left(p_t V_{k_{t+1}}^{t+1} + V_{Q_{t+1}}^{t+1}\frac{\partial P_t}{\partial g_{t,2}}\right), \tag{3.A.12}$$

$$a_t: \quad \beta V_{a_t\delta_t}^{t+1}\left(\delta_t + a_t\frac{\partial H}{\partial a_t}\right) = 0, \tag{3.A.13}$$

$$l_t^g: \quad \beta\left[V_{k_{t+1}}^{t+1}\frac{\partial G}{\partial l_t^g} - V_{a_t\delta_t}^{t+1}a_t\frac{\partial H}{\partial(l_t - l_t^g)}\right] = 0, \tag{3.A.14}$$

$$l_t: \quad U_{1-l_t} = \beta V_{a_t\delta_t}^{t+1}a_t\frac{\partial H}{\partial(l_t - l_t^g)}, \tag{3.A.15}$$

$$k_t^g: \quad V_{k_{t+1}}^{t+1}\left(\frac{\partial G}{\partial k_t^g}\right) - V_{a_t\delta_t}^{t+1}\left[a_t\frac{\partial H}{\partial(k_t - k_t^g)}\right] = 0. \tag{3.A.16}$$

Envelope conditions:

$$V_{k_t}^t = \beta V_{k_{t+1}}^{t+1}(1-\epsilon_k) + \beta V_{a_t\delta_t}^{t+1}a_t\frac{\partial H}{\partial(k_t - k_t^g)}, \tag{3.A.17}$$

$$V_{Q_t}^t = U_Q + \beta V_{Q_{t+1}}^{t+1}(1+\epsilon), \tag{3.A.18}$$

$$V_{(a_{t-1}\delta_{t-1})}^t = U_{M_t}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial(a_{t-1}\delta_{t-1})} + U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial(a_{t-1}\delta_{t-1})} + \rho\beta V_{(\rho a_{t-1}\delta_{t-1})}^{t+1}, \tag{3.A.19}$$

$$V_{(\rho a_{t-2}\delta_{t-2})}^t = U_{M_t}\frac{\partial M_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial(\rho a_{t-2}\delta_{t-2})} + U_{N_t}\frac{\partial N_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial(\rho a_{t-2}\delta_{t-2})}. \tag{3.A.20}$$

3.9.3 Constrained social planner's problem

We need to guarantee that the first order conditions and envelope conditions measure the same marginal changes for social planner under the gasoline constraint. We set up a constrained social planner problem to see whether the marginal changes match with what we come up with above.

The objective function for social planner is the same with Eq.3.4.26:

$$V^t(k_t, Q_t; a_{t-1}\delta_{t-1}, \rho a_{t-2}\delta_{t-2}; \{I_t\}) =$$

$$\max_{c_t, g_{t,1}, g_{t,2}, a_t, l_t^g, l_t, k_t^g, k_t} \Big[U(c_t, M_t, 1-l_t, N_t, Q_t) + \beta V^{t+1}(k_{t+1}, Q_{t+1}; a_t\delta_t, \rho a_{t-1}\delta_{t-1}; \{I_{t+1}\})\Big], \tag{3.A.21}$$

subject to:

$$G(k_t^g, l_t^g) = c_t + k_{t+1} - (1-\epsilon_k)k_t + p_t(g_{t,1} + g_{t,2}), \tag{3.A.22}$$

$$F(k_t^a, l_t^a) = a_t + \mu\delta_t, \tag{3.A.23}$$

$$Q_{t+1} - Q_t = \Phi - \epsilon Q_t - P_t, \tag{3.A.24}$$

$$g_{t,1} = \Psi(a_{t-1}\delta_{t-1}, \rho a_{t-1}\delta_{t-2})g_{t,2}. \tag{3.A.25}$$

Thus, the corresponding first-order conditions are:

$$c_t: \quad U_{c_t} = \beta V_{k_{t+1}}^{t+1}, \tag{3.A.26}$$

$$\begin{aligned} g_{t,2}: \quad & U_{M_t}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}}\frac{\partial g_{t,1}}{\partial g_{t,2}} + U_{M_t}\frac{\partial M_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial g_{t,2}} + U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}}\frac{\partial g_{t,1}}{\partial g_{t,2}} \\ & + U_{N_t}\frac{\partial N_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial g_{t,2}} = \beta\left[V_{k_{t+1}}^{t+1}p_t(\Psi+1) + V_{Q_{t+1}}^{t+1}\left(\frac{\partial P_t}{\partial g_{t,2}} + \frac{\partial P_t}{\partial g_{t,1}}\frac{\partial g_{t,1}}{\partial g_{t,2}}\right)\right], \end{aligned} \tag{3.A.27}$$

$$a_t: \quad \beta V_{a_t\delta_t}^{t+1}\left(\delta_t + a_t\frac{\partial H}{\partial a_t}\right) = 0, \tag{3.A.28}$$

$$l_t^g: \quad \beta\left[V_{k_{t+1}}^{t+1}\frac{\partial G}{\partial l_t^g} - V_{a_t\delta_t}^{t+1}a_t\frac{\partial H}{\partial(l_t - l_t^g)}\right] = 0, \tag{3.A.29}$$

$$l_t: \quad U_{1-l_t} = \beta V_{a_t\delta_t}^{t+1}a_t\frac{\partial H}{\partial(l_t - l_t^g)}, \tag{3.A.30}$$

$$k_t^g: \quad \beta V_{k_{t+1}}^{t+1}\frac{\partial G}{\partial k_t^g} - \beta V_{a_t\delta_t}^{t+1}a_t\frac{\partial H}{\partial(k_t - k_t^g)} = 0. \tag{3.A.31}$$

And the envelope conditions:

$$V_{k_t}^t = \beta V_{k_{t+1}}^{t+1}(1-\epsilon_k) + \beta V_{a_t\delta_t} a_t \frac{\partial H}{\partial(k_t - k_t^g)}, \tag{3.A.32}$$

$$V_{Q_t}^t = U_{Q_t} + \beta V_{Q_{t+1}}^{t+1}(1+\epsilon), \tag{3.A.33}$$

$$\begin{aligned} V_{(a_{t-1}\delta_{t-1})}^t &= U_{M_t}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial(a_{t-1}\delta_{t-1})} + U_{M_t}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}}\frac{\partial g_{t,1}}{\partial(a_{t-1}\delta_{t-1})} \\ &+ U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial(a_{t-1}\delta_{t-1})} + U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}}\frac{\partial g_{t,1}}{\partial(a_{t-1}\delta_{t-1})} \\ &+ \rho\beta V_{\rho a_{t-1}\delta_{t-1}}^{t+1} - \beta V_{k_{t+1}}^{t+1} p_t \frac{\partial g_{t,1}}{\partial(a_{t-1}\delta_{t-1})} - \beta V_{Q_{t+1}}^{t+1}\frac{\partial P_t}{\partial g_{t,1}}\frac{\partial g_{t,1}}{\partial(a_{t-1}\delta_{t-1})}, \end{aligned} \tag{3.A.34}$$

$$\begin{aligned} V_{(\rho a_{t-2}\delta_{t-2})}^t &= U_{M_t}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}}\frac{\partial g_{t,1}}{\partial(\rho a_{t-2}\delta_{t-2})} + U_{M_t}\frac{\partial M_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial(\rho a_{t-2}\delta_{t-2})} \\ &+ U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial(\rho a_{t-2}\delta_{t-2})} + U_{N_t}\frac{\partial N_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial(\rho a_{t-2}\delta_{t-2})} \\ &- \beta V_{k_{t+1}}^{t+1} p_t \frac{\partial g_{t,1}}{\partial(\rho a_{t-2}\delta_{t-2})} - \beta V_{Q_{t+1}}^{t+1}\frac{\partial P_t}{\partial g_{t,1}}\frac{\partial g_{t,1}}{\partial(\rho a_{t-2}\delta_{t-2})}. \end{aligned} \tag{3.A.35}$$

The envelope conditions with respect to $a_{t-1}\delta_{t-1}$, $\rho a_{t-2}\delta_{t-2}$ match with Eq. 3.6.37 and Eq.3.6.38.

3.9.4 Steady state solution

Equations describing the economy in the steady states are:

$$A_1(k^g)^{\alpha_1}(l^g)^{1-\alpha_1} = c + \epsilon_k k + p_t(g_1+g_2), \tag{3.A.36}$$

$$k^a + k^g = k, \tag{3.A.37}$$

$$l^a + l^g = l, \tag{3.A.38}$$

$$\mu\delta = A_2(k^a)^{\alpha_2} l^{a\frac{1}{2}-\alpha_2} - a, \tag{3.A.39}$$

$$\epsilon Q = \Phi - P, \tag{3.A.40}$$

and

$$V_k = \frac{U_c}{\beta}, \tag{3.A.41}$$

$$U_M \frac{\partial M}{\partial m_1}\frac{\partial m_1}{\partial g_1} + U_N \frac{\partial N}{\partial m_1}\frac{\partial m_1}{\partial g_1} = \beta(p_t V_k + V_Q \frac{\partial P}{\partial g_1}), \tag{3.A.42}$$

$$U_M \frac{\partial M}{\partial m_2}\frac{\partial m_2}{\partial g_2} + U_N \frac{\partial N}{\partial m_2}\frac{\partial m_2}{\partial g_2} = \beta(p_t V_k + V_Q \frac{\partial P}{\partial g_2}), \tag{3.A.43}$$

$$a = \mu\delta, \tag{3.A.44}$$

$$V_k \frac{\partial G}{\partial l^g} = V_{a\delta} a \frac{\partial H}{\partial l^a}, \tag{3.A.45}$$

$$U_{1-l} = \beta V_{a\delta} a \frac{\partial H}{\partial l^a}, \tag{3.A.46}$$

$$V_k \frac{\partial G}{\partial k^g} = V_{a\delta} a \frac{\partial H}{\partial k^a}, \tag{3.A.47}$$

$$V_k = \beta V_k(1-\epsilon_k) + \beta V_{a\delta} a \frac{\partial H}{\partial k^a}, \tag{3.A.48}$$

$$V_Q = U_Q + \beta V_Q(1+\epsilon), \tag{3.A.49}$$

$$V_{a\delta} = U_M \frac{\partial M}{\partial m_1}\frac{\partial m_1}{\partial (a\delta)} + U_N \frac{\partial N}{\partial m_1}\frac{\partial m_1}{\partial (a\delta)} + \rho\beta V_{\rho a\delta}, \tag{3.A.50}$$

$$V_{\rho a\delta} = U_M \frac{\partial M}{\partial m_2}\frac{\partial m_2}{\partial (\rho a\delta)} + U_N \frac{\partial N}{\partial m_2}\frac{\partial m_2}{\partial (\rho a\delta)}. \tag{3.A.51}$$

Using the marginal substitution between consumption and capital, we can get rid of V_k. We then have the marginal substitution between consumption and labour:

$$U_c \frac{\partial G}{\partial k^g} = U_{1-l}. \tag{3.A.52}$$

The capital labour ratio between the two production sectors is:

$$\frac{\frac{\partial G}{\partial l^g}}{\frac{\partial G}{\partial k^g}} = \frac{\frac{\partial H}{\partial l^a}}{\frac{\partial H}{\partial l^a}}. \tag{3.A.53}$$

The marginal productivity of labour in general production function is expressed as:

$$\frac{\partial G}{\partial k^g} = \frac{1-\beta(1-\epsilon_k)}{\beta}. \tag{3.A.54}$$

We get the steady state conditions:

$$A_1(\frac{k^g}{l^g})^{\alpha_1}l^g = c + \epsilon_k k + p_t(g_1 + g_2)$$

$$k^a + k^g = k$$

$$l^a + l^g = l$$

$$\mu\delta = A_2(\frac{k^a}{l^a})^{\alpha_2}(l^a)^{\frac{1}{2}} - a$$

$$\epsilon Q = \Phi - P$$

$$P = \frac{g_1}{\delta} + \frac{g_2}{\rho\delta}$$

$$\frac{\Phi_2 g_1^{\sigma-1}}{g_1^\sigma + \rho^{\gamma\sigma}g_2^\sigma} + \frac{\Phi_3}{g_1 + \rho^\gamma g_2} = \frac{p_t\Phi_1}{c} + \frac{\beta}{1-\beta(1+\epsilon)}\frac{1}{\delta}\frac{\Phi_4}{Q}$$

$$\frac{\Phi_2\rho^{\gamma\sigma}g_2^{\sigma-1}}{g_1^\sigma + \rho^{\gamma\sigma}g_2^\sigma} + \frac{\Phi_3\rho^\gamma}{g_1 + \rho^\gamma g_2} = \frac{p_t\Phi_1}{c} + \frac{\beta}{1-\beta(1+\epsilon)}\frac{1}{\rho\delta}\frac{\Phi_4}{Q}$$

$$a = \mu\delta$$

$$(\frac{k^g}{l^g})^{\alpha_1} = \frac{1-\Phi_1-\Phi_2}{\Phi_1 A_1(1-\alpha_1)}\frac{c}{1-l}$$

$$\frac{k^a}{l^a} = \frac{1-\alpha_1}{\alpha_1}\frac{\alpha_2}{\frac{1}{2}-\alpha_2}\frac{k^g}{l^g}$$

$$\frac{k^g}{l^g} = \left[\frac{1-\beta(1-\epsilon_k)}{\beta A_1\alpha_1}\right]^{\frac{1}{\alpha_1-1}}$$

$$\frac{[1-\beta(1-\epsilon_k)]\mu}{\beta A_2\alpha_2(\frac{1-\alpha_1}{\alpha_1})^{\alpha_2-\frac{1}{2}}(\frac{\alpha_2}{\frac{1}{2}-\alpha_2})^{\alpha_2-\frac{1}{2}}(\frac{1-\beta(1-\epsilon_k)}{\beta A_1\alpha_1})^{\frac{\alpha_2-\frac{1}{2}}{\alpha_1-1}}(k^a)^{-\frac{1}{2}}}\frac{\Phi_1}{\beta c} = \frac{p_t\Phi_1(g_1+\beta g_2)}{c} + \frac{\beta\Phi_4(g_1+\beta g_2)}{(1-\beta(1+\epsilon))\delta Q}$$

3.9.5 Numerical results

Table 3.A.1 describes the economy when both externalities from driving are considered and the preference for environment varies.

Table 3.A.2 describes the uniform fuel tax case when both congestion and pollution are considered and the preference for environment varies.

Scenario 3: pollution and congestion (ϕ_4 varies)

Variable	Description	Value			
		$\phi_4 = 0$	$\phi_4 = 0.1$	$\phi_4 = 0.34$	$\phi_4 = 1$
Economy in the steady state					
c	consumption	0.3677	0.3682	0.3694	0.3720
g_1	gasoline (new cars)	0.0254	0.0247	0.0231	0.0197
g_2	gasoline (old cars)	0.0241	0.0233	0.0215	0.0177
k^g	capital (general production)	1.4767	1.4729	1.4647	1.4472
k^a	capital (vehicle production)	0.1654	0.1657	0.1663	0.1675
k	total capital	1.6422	1.6387	1.6310	1.6148
a	vehicle capital	0.1507	0.1508	0.1511	0.1516
δ	vehicle efficiency	0.1507	0.1508	0.1511	0.1516
l^a	labour (vehicle production)	0.003906	0.003913	0.003926	0.003955
l^g	labour (general production)	0.3716	0.3706	0.3685	0.3641
l	total labour	0.3755	0.3745	0.3725	0.3681
P	pollution	0.3471	0.3358	0.3114	0.2597
Optimal Environmental Taxation					
τ^1	optimal fuel tax (new cars)	0.0021	0.0376	0.1227	0.3561
τ^2	optimal fuel tax (old cars)	0.0020	0.0414	0.1360	0.3953
T_1	optimal road tax (new cars)	0.0011691	0.001136	0.001064	9.0788e-04
T_2	optimal road tax (old cars)	0.0011692	0.001128	0.001041	8.6048e-04
Mileage of Travel					
m_1	mileage travel (new cars)	0.0038	0.0037	0.0035	0.0030
m_2	mileage travel (old cars)	0.0035	0.0033	0.0031	0.0025
Travel Cost					
$(p_t + \tau^1)g_1$	gasoline cost for new cars	0.0277	0.0278	0.0280	0.0284
$(p_t + \tau^2)g_2$	gasoline cost for old cars	0.0263	0.0263	0.0263	0.0263
$T_1(a\delta)$	road tax cost for new cars	2.6562e-05	2.5861e-05	2.4318e-05	2.0884e-05
$T_2(a\delta)$	road tax cost for old cars	2.6566e-05	2.5685e-05	2.3792e-05	1.9793e-05
q^a	vehicle price	1.135329	1.135328	1.135334	1.135340
$q^a a\delta$	vehicle purchase cost	0.025794	0.025835	0.025925	0.02611

Table 3.A.1: Optimal environmental taxes and economy when both externalities considered: preference for environment varies

Uniform Tax: pollution and congestion (ϕ_4 varies)					
Economy in the steady state					
Variable	**Description**	**Value**			
		$\phi_4 = 0$	$\phi_4 = 0.1$	$\phi_4 = 0.34$	$\phi_4 = 1$
c	consumption	0.3677	0.3682	0.3694	0.3720
g_1	gasoline (new cars)	0.0254	0.0246	0.0229	0.0191
g_2	gasoline (old cars)	0.0241	0.0234	0.0217	0.0182
k_g	capital (general production)	1.4767	1.4729	1.4647	1.4472
k_a	capital (vehicle production)	0.1654	0.1657	0.1663	0.1675
k	total capital	1.6422	1.6386	1.6310	1.6147
a	vehicle capital	0.1507	0.1508	0.1511	0.1516
δ	vehicle efficiency	0.1507	0.1508	0.1511	0.1516
l_a	labour (vehicle production)	0.003906	0.003913	0.003926	0.003954
l_g	labour (general production)	0.3716	0.3706	0.3685	0.3641
l	total labour	0.3755	0.3745	0.3725	0.3681
P	pollution	0.3471	0.3358	0.3116	0.2600
Optimal Environmental Taxation					
τ	optimal fuel tax (new cars)	0.0020	0.0395	0.1292	0.3752
T_1	optimal road tax (new cars)	0.0012	0.00113	0.00105	8.8456e-04
T_2	optimal road tax (old cars)	0.0012	0.00113	0.00105	8.8456e-04
Mileage of Travel					
m_1	mileage travel(new cars)	0.0038	0.0037	0.0035	0.0029
m_2	mileage travel (old cars)	0.0035	0.0033	0.0031	0.0026
Travel Cost					
$(p_t + \tau^1)g_1$	gasoline cost (new cars)	0.0277	0.0278	0.0279	0.0281
$(p_t + \tau^2)g_2$	gasoline cost (old cars)	0.0263	0.0264	0.0265	0.0266
$T_1(a\delta)$	road tax cost (new cars)	2.6564e-05	2.5774e-05	2.4059e-05	2.0343e-05
$T_2(a\delta)$	road tax cost (old cars)	2.6564e-05	2.5774e-05	2.4059e-05	2.0343e-05
q^a	vehicle price	1.13533	1.135329	1.135337	1.135342
$q^a a\delta$	vehicle purchase cost	0.025794	0.025835	0.025923	0.02611

Table 3.A.2: Uniform fuel tax, corresponding road taxes and the economy with both types of externalities: preference for environment varies

Chapter 4

Optimal Environmental Taxes in the Presence of Distortionary Taxes

In this chapter, we derive the optimal environmental tax (gasoline taxes and road use taxes) structure for vehicles of different vintages in the presence of distortionary taxes and externalities caused by vehicle driving (pollution and congestion). The analytical results from our model show the additive property between the Pigovian element and the efficiency element in the formulation of optimal gasoline taxes and optimal road taxes. We also show that the optimal environmental taxes depend on the households' preferences for environmental quality and congestion externality. The optimal tax structure is also determined by the degree of complementarity with common consumption good.

4.1 Introduction

In this chapter, we derive the optimal environmental tax (gasoline taxes and road use taxes) structure for vehicles of different vintages (new cars and old cars) in the presence of distortionary taxes (labour tax, capital income tax and vehicle purchase tax) and externalities caused by vehicle driving (pollution and congestion). Households own both new cars and old cars, which provide them with driving services. However, vehicle driving leads to pollution and congestion externalities which impose a negative impact on households' overall happiness level.

Previous literature on optimal environmental taxes focuses on how to apply one type of environmental tax to address many environmental externalities. Parry and Small (2005) derive the optimal gasoline tax formula in the second best considering externalities caused by vehicle driving (pollution, congestion and accidents). Based on the optimal gasoline tax formulation, their simulation results show that the gasoline tax for the U.S. is too low while too high for the U.K. Bovenberg and Goulder (1996) examines how optimal environmental tax rates deviate from rates implied by the Pigovian principle in a second-best setting with the presence of other distortionary taxes where environmental tax is applied to "dirty" intermediate production inputs. They find that in the presence of distortionary taxes, optimal environmental tax rates are generally below the rates suggested by the Pigovian principle. In this chapter, we broaden the analysis to include different environmental taxes (gasoline taxes and road taxes) to see their interrelation when other distortionary taxes are present. By doing so, we can examine how the presence of distortionary taxes affect the structure of optimal environmental taxes.

We approach the problem by taking the perspectives from both the individual and the government. Disintegrating the optimal environmental taxes, we find the additive property between the Pigovian element and efficient element proposed by Sandmo (1975). The presence of distortionary taxes cause optimal environmental taxes deviating from the Pigovian standards and the deviation depends on household's preference for environmental quality and congestion externality.

To further examine the results of our optimal environmental taxes, we apply the approach proposed by Atkinson and Stiglitz (1972) and look into how optimal tax structure can be explained by complementarity with normal consumption goods when utility function is not direct additive. The formulation of optimal gasoline taxes depends on two opposing factors: the marginal cost of pollution and the marginal cost of congestion. Whether one factor outweighs the other depends on both their degree of complementarity with the normal consumption goods and households' preference on environmental factors.

This chapter is organized as follows. Section 4.2 first introduces the decentralized economy where individual household does not internalize the detrimental effects caused by vehicle driving to the environment. We then present household's problem. Section 4.3 formulates the problem from the government's perspective (the Ramsey problem). Section 4.4 presents the optimal tax structure and the implications of the tax structure are discussed in section 4.5. Section 4.6 concludes.

4.2 The economy

4.2.1 Assumptions about production

The economy is constituted by two sectors: the general production sector G and the vehicle production sector F. Both sectors require capital k and labour l as production inputs.

General production: In this sector, firms hire labour l_t^g and rent capital k_t^g from households at the price of r_t^g and w_t^g to produce consumption goods c, accumulate capital k, and import gasoline (g_1 and g_2) at a constant price p_t with constant-return-to-scale technology:

$$G(k_t^g, l_t^g) = c_t + k_{t+1} - (1 - \epsilon_k)k_t + p_t(g_{t,1} + g_{t,2}). \tag{4.2.1}$$

The problem facing firms in the general production sector is to choose capital and labour to maximize profits π_t^g:

$$\max_{k_t^g, l_t^g} \pi_t^g = G(k_t^g, l_t^g) - r_t^g k_t^g - w_t^g l_t^g. \tag{4.2.2}$$

The first-order conditions then read:

$$G_{k_t^g} = r_t^g, \tag{4.2.3}$$

$$G_{l_t^g} = w_t^g. \tag{4.2.4}$$

Vehicle production: In this sector, firms hire labour l_t^a and rent capital k_t^a to produce vehicle capital a_t and fuel efficiency δ_t:

$$F(k_t^a, l_t^a) = a_t + \mu\delta_t \tag{4.2.5}$$

Vehicle is a type of capital good which is made up of two attributes (a and δ). a is vehicle capital and δ measures the fuel efficiency level embedded within the vehicles. Those two components are produced separately but must be sold as a combined product at the price of q_t^a. Firms in this sector choose labour l_t^a, capital k_t^a and the optimal combination of $a_t\delta_t$ to maximize the profits π_t^a:

$$\max_{k_t^a, l_t^a, a_t\delta_t} \pi_t^a = q_t^a(a_t\delta_t) - r_t^a k_t^a - w_t^a l_t^a. \tag{4.2.6}$$

The first-order conditions are:

$$r_t^a = q_t^a \delta_t F_{k_t^a}, \tag{4.2.7}$$

$$w_t^a = q_t^a \delta_t F_{l_t^a}, \tag{4.2.8}$$

$$q_t^a[F(k_t^a, l_t^a) - \mu\delta_t] - \mu q_t^a \delta_t = 0. \tag{4.2.9}$$

Equilibrium in production: Market clearing implies:

$$k_t^a + k_t^g = k_t, \tag{4.2.10}$$

$$l_t^g + l_t^a = l_t, \tag{4.2.11}$$

where l_t denotes the total labour and k_t total capital at time period t.

$$w_t^a = w_t^g = w_t, \tag{4.2.12}$$

$$r_t^a = r_t^g = r_t. \tag{4.2.13}$$

4.2.2 Assumptions about the households

Representative household gains utility from general consumption c_t, driving service M_t, leisure $1 - l_t$ and environment quality Q_t. Household gets disutility from congestion N_t:

$$U(c_t, M_t, 1 - l_t, N_t, Q_t). \tag{4.2.14}$$

Driving behavior: There are two types of vehicles in the market: new cars and old cars. We follow Solow et al. (1960b) and Cooley et al. (1997) to model vintage vehicles using capital heterogeneity as in previous chapters. After production, the technology embedded in the vehicle will not change, which implies that the mileage of travel over one unit of gasoline consumed is fixed for the specific vintage type. Vehicles need one period of configuration and are then used by the households for two periods before getting scraped. New cars produced at time period t are used by the households at time period $t+1$. At time period $t+2$, new cars become old cars and they are also subject to depreciation $(1-\rho)$ from already being used for a time period.

The mileage of travel produced by both new $(m_{t,1})$ and old $(m_{t,2})$ cars constitute driving service in the following way[1]:

$$M_t = (m_{t,1}^{\sigma} + m_{t,2}^{\sigma})^{\frac{1}{\sigma}}, \tag{4.2.15}$$

where $0 < \sigma < 1$ and it measures price elasticity of demand.

We further specify the mileage of travel provided by the new cars and the old

[1]The preferences for $m_{t,1}$ and $m_{t,2}$ follow Grossman and Helpman (1991) to guarantee that household exhibits preference for variety over quantity, which means that household always prefers to use both types of cars instead of just using new cars.

cars following Wei (2013):

$$m_{t,1} = (a_{t-1}\delta_{t-1})^{\gamma} g_{t,1}, \tag{4.2.16}$$

$$m_{t,2} = (\rho a_{t-2}\delta_{t-2})^{\gamma} g_{t,2}, \tag{4.2.17}$$

where $0 < \rho < 1$ and $0 < \gamma < 1$.

Eq.4.2.16 and Eq.4.2.17 show that mileage of travel is linearly related to gasoline consumed by different vehicles. γ measures the technology embedded in the vehicle after production.

Congestion: One important byproduct of vehicle driving is congestion. Congestion is normally modelled as the time spent on driving [2]. We assume that the average speed of people spend on driving is an exogenous constant[3] (Parry and Small, 2005). Thus, we could use the sum of mileage as a proxy for congestion N_t.

$$N_t = m_{t,1} + m_{t,2}. \tag{4.2.18}$$

Pollution and environmental quality: Household gains utility from good environment quality. Gasoline combustion caused by vehicle driving causes pollution which is mitigated by more fuel-efficient vehicles. Pollution at each period (P_t) is positively related to gasoline consumption while mitigated by the fuel efficiency conditions embedded in different vintage vehicles (Selden and Song, 1995):

$$P_t = \frac{g_{t,1}}{\delta_{t-1}} + \frac{g_{t,2}}{\rho\delta_{t-2}}, \tag{4.2.19}$$

[2] See Arnott and Small (1994) and Rouwendal and Verhoef (2006).

[3] Households do not take account of their own impact on congestion.

where $0 < \rho < 1$ and $1-\rho$ measures the depreciation to the old cars for having been used for a period.

Environmental quality is modelled as a type of capital asset. The quality of the environment, Q represents the stock of natural capital and accumulates based on the regenerating ability of nature while depreciating due to pollution P. Environmental quality evolves over time according to the following function based on Bovenberg and Smulders (1995):

$$Q_{t+1} - Q_t = R - \epsilon Q_t - P_t, \tag{4.2.20}$$

where $Q_{max} = \bar{Q}$. R represents the original status of the environmental quality and ϵ measures the nature's pollutant-assimilating ability. Environmental quality Q can not explode thus we assume an upper limit Q_{max} for it.

4.2.3 Household's problem

Infinitely-lived representative household supplies labour l_t and capital k_t to firms at wage rate w_t and capital rental price r_t, and their income are subject to labour tax τ_t^l and capital income tax τ_t^k. It also receives the profits generated from both production sectors (π_t^g and π_t^a). Household purchases consumption goods (c_t), gasoline ($g_{t,1}$, $g_{t,2}$), new vehicles ($a_t\delta_t$) and make investments. Household is also subject to gasoline taxes (τ_t^1 for new cars and τ_t^2 for old cars), road taxes (T_t^1 for new cars and T_t^2 for old cars) and vehicle purchase tax (τ_t^a).

Household maximizes its life-time utility:

$$\max_{c_t, g_{t,1}, g_{t,2}, a_t\delta_t, l_t} \sum_{s=0}^{\infty} \beta^{t+s} U(c_t, M_t, 1-l_t, N_t, Q_t), \tag{4.2.21}$$

subject to the budget constraint:

$$\pi_t^a + \pi_t^g + (1-\tau_t^l)w_t l_t + \left[1-\epsilon+(1-\tau_t^k)r_t\right]A_t =$$

$$(p_t+\tau_t^1)g_{t,1}+(p_t+\tau_t^2)g_{t,2}+A_{t+1}+c_t+(1+\tau_t^a)q_t^a(a_t\delta_t)+T_{t,1}(a_{t-1}\delta_{t-1})+T_{t,2}(a_{t-2}\delta_{t-2}), \tag{4.2.22}$$

where A_t denotes the total assets owned by the household and it consists of capital k_t and government bonds B_t ($A_t = k_t + B_t$).

The Lagrangian reads:

$$\mathcal{L} = \sum_{t=0}^{\infty}\beta^t\Big[U(c_t, M_t, 1-l_t, N_t, Q_t) + \lambda_t\Big(\pi_t^a + \pi_t^g + (1-\tau_t^l)w_t l_t + [1-\epsilon_k+(1-\tau_t^k)r_t]A_t$$
$$-(p_t+\tau_t^1)g_{t,1} - (p_t+\tau_t^2)g_{t,2} - A_{t+1} - c_t - (1+\tau_t^a)q_t^a(a_t\delta_t)$$
$$-T_{t,1}(a_{t-1}\delta_{t-1}) - T_{t,2}(a_{t-2}\delta_{t-2})\Big)\Big]. \tag{4.2.23}$$

Solving the Lagrangian problem (see Appendix 4.7.1), we get the following first-order conditions:

$$\frac{U_{1-l_t}}{U_{c_t}} = (1-\tau_t^l)w_t, \tag{4.2.24}$$

$$\frac{U_{c_{t-1}}}{\beta U_{c_t}} = \left[1-\epsilon+(1-\tau_t^k)r_t\right], \tag{4.2.25}$$

$$\frac{\frac{\partial U_t}{\partial M_t}\frac{\partial M_t}{\partial m_{t,i}}\frac{\partial m_{t,i}}{\partial g_{t,i}}}{U_{c_t}} = (p_t+\tau_t^i), \tag{4.2.26}$$

where $i = 1, 2$.

Eq.4.2.24 and Eq.4.2.26 describe the marginal rate of substitution between leisure, gasoline consumption and general consumption goods. Eq.4.2.25 determines that capital rental price.

4.3 The Ramsey problem

We employ the primal approach which enables us to maximize the social welfare directly through choices of allocations [4].

The implementability constraint reads can be obtained by rearranging budget constraint Eq.4.2.22[5],:

$$0 = \lambda_{-1} A_0 + \sum_{t=0}^{\infty} \beta^t \Big(U_{1-l_t} l_t - U_{M_t} M_t - U_{c_t} c_t - \gamma \beta U_{M_{t+1}} M_{t+1} \Big). \tag{4.3.27}$$

With the implementability constraint, we present the government's problem: government maximizes the social welfare with respect to the resource constraint (Eq.4.2.1).

The Lagrangian reads:

$$\begin{aligned} \mathcal{L}^g = \sum_{t=0}^{\infty} \beta^t \Bigg[& U(c_t, M_t, 1 - l_t, N_t, Q_t) + \tilde{\Omega} \Big(U_{l_t} l_t + U_{M_t} M_t + U_{c_t} c_t + \gamma \beta U_{M_{t+1}} M_{t+1} \Big) \\ & + \Psi_t \Big(G(k_t^g, l_t^g) - c_t + (1 - \epsilon_k) k_t - k_{t+1} - p_t (g_{t,1} + g_{t,2}) \Big) \\ & + \Phi_t \Big((1 - \epsilon) Q_t + R - P_t - Q_{t+1} \Big) \Bigg], \end{aligned} \tag{4.3.28}$$

where $\tilde{\Omega}$ and Ψ_t are Lagrange multipliers. $\tilde{\Omega}$ measures the effect of an increase in tax rate on social utility while Ψ_t measures the effect of income change on social utility.

We derive the first-order conditions with respect to consumption c_t, labour (l_t^a, l_t), gasoline consumption ($g_{t,1}$, $g_{t,2}$), capital (k_t^a, k_t) and environmental quality(Q_t) for the next period[6].

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[4] See Atkinson and Stiglitz (2015).

[5] The detailed derivation can be found in Appendix 4.7.2.

[6] The detailed derivation can be found in Appendix 4.7.3.

Simplifying the first-order conditions with respect to consumption (See Eq.4.A.17) and labour (See Eq.4.A.18) obtained in the government's problem, we get:

$$1 + \tilde{\Omega}\Delta_{c_t} = \frac{\Psi_t}{U_{c_t}}, \tag{4.3.29}$$

$$1 + \tilde{\Omega}\Delta_{l_t} = -\frac{\Psi_t G_{l_t^g}}{U_{l_t}}, \tag{4.3.30}$$

where

$$\Delta_{c_t} = 1 + \frac{U_{cc}c_t}{U_{c_t}} + \frac{U_{lc}l_t}{U_{c_t}} + (1+\gamma)\frac{U_{Mc}M_t}{U_{c_t}},$$

$$\Delta_{l_t} = 1 + \frac{U_{ll}l_t}{U_{l_t}} + \frac{U_{cl}c_t}{U_{l_t}} + (1+\gamma)\frac{U_{Ml}M_t}{U_{l_t}}.$$

Then, we divide Eq. 4.3.29 by Eq.4.3.30 to obtain the following:

$$\frac{1 + \tilde{\Omega}\Delta_{c_t}}{1 + \tilde{\Omega}\Delta_{l_t}} = -\frac{U_{l_t}}{U_{c_t}}\frac{1}{G_{l_t^g}}.$$

Using Eq.4.2.24, we get the optimal labour tax rate τ_t^l:

$$\tau_t^l = 1 - \frac{1 + \tilde{\Omega}\Delta_{c_t}}{1 + \tilde{\Omega}\Delta_{l_t}} = \frac{\tilde{\Omega}\Delta_{l_t} - \tilde{\Omega}\Delta_{c_t}}{1 + \tilde{\Omega}\Delta_{l_t}}. \tag{4.3.31}$$

In the steady state, consumption c, labour l and driving service M are constant which means that the Lagrangian multiplier Ψ is constant in the long run. Therefore, in equilibrium, the first-order condition with respect to capital (See Eq.4.A.19) becomes:

$$1 = \beta(G_{k_t^g} + 1 - \epsilon_k). \tag{4.3.32}$$

Combining this result with Eq.4.2.25, the optimal capital tax in the steady state is zero which is consistent with the results by Ramsey (1928):

$$\tau^k = 0. \tag{4.3.33}$$

4.4 Optimal environmental taxes

Vehicle driving causes pollution and congestion externalities. Given that vehicle driving service is generated by two components (transportation capital and gasoline consumption), the optimal gasoline taxes and the optimal road taxes should reflect this feature.

Optimal gasoline taxes

Gasoline is used by two types of vehicles which provide driving services to the household at each time period. Given that the optimal amount of gasoline chosen by the government depends on the Lagrangian multiplier Φ_t, we look at the scenario at the steady state. In the steady state, the first order conditions with respect to gasoline consumption in the government's problem become[7]:

$$U_M(1+\tilde{\Omega}\Delta_M)M^{1-\sigma}\frac{m_1^\sigma}{g_1}+U_N(1+\tilde{\Omega}\Delta_N)\frac{m_1}{g_1}-\Phi\frac{1}{\delta}=\Psi p_t,$$
$$U_M(1+\tilde{\Omega}\Delta_M)M^{1-\sigma}\frac{m_2^\sigma}{g_2}+U_N(1+\tilde{\Omega}\Delta_N)\frac{m_2}{g_2}-\Phi\frac{1}{\rho\delta}=\Psi p_t.$$

Given the steady state value of Φ (See Eq.4.A.25), we obtain:

$$\frac{U_M M^{1-\sigma}}{U_c}\frac{m_1^\sigma}{g_1}-p_t=-\frac{U_N}{U_c}\frac{(1+\tilde{\Omega}\Delta_N)}{(1+\tilde{\Omega}\Delta_M)}\frac{m_1}{g_1}+\frac{1}{\delta}\frac{U_Q}{U_c}\frac{(1+\tilde{\Omega}\Delta_Q)}{1+\tilde{\Omega}\Delta_M}\frac{\beta}{1-\beta(1-\epsilon)},$$
$$\frac{U_M M^{1-\sigma}}{U_c}\frac{m_2^\sigma}{g_2}-p_t=-\frac{U_N}{U_c}\frac{(1+\tilde{\Omega}\Delta_N)}{(1+\tilde{\Omega}\Delta_M)}\frac{m_2}{g_2}+\frac{1}{\rho\delta}\frac{U_Q}{U_c}\frac{(1+\tilde{\Omega}\Delta_Q)}{1+\tilde{\Omega}\Delta_M}\frac{\beta}{1-\beta(1-\epsilon)}.$$

[7] Firms' profit maximizing decisions imply that:

$$\frac{G_k}{G_l}=\frac{F_{k^a}}{F_{l^a}}.$$

Using Eq. 4.2.26, the optimal gasoline taxes in the steady state are expressed by:

$$\tau^1 = -\frac{U_N}{U_c}\frac{(1+\tilde{\Omega}\Delta_N)}{(1+\tilde{\Omega}\Delta_M)}\frac{m_1}{g_1} + \frac{1}{\delta}\frac{U_Q}{U_c}\frac{(1+\tilde{\Omega}\Delta_Q)}{(1+\tilde{\Omega}\Delta_M)}\frac{\beta}{1-\beta(1-\epsilon)}, \tag{4.4.34}$$

$$\tau^2 = -\frac{U_N}{U_c}\frac{(1+\tilde{\Omega}\Delta_N)}{(1+\tilde{\Omega}\Delta_M)}\frac{m_2}{g_2} + \frac{1}{\rho\delta}\frac{U_Q}{U_c}\frac{(1+\tilde{\Omega}\Delta_Q)}{(1+\tilde{\Omega}\Delta_M)}\frac{\beta}{1-\beta(1-\epsilon)}. \tag{4.4.35}$$

Optimal road taxes

Total mileage of travel $(m_{t,1} + m_{t,2})$ is used as a proxy for the congestion. The mileages of travel by both new cars and old cars depend on the vintages of vehicles and the amount of gasoline they consumed. $a_t\delta_t$ is determined in the government's resource allocation problem which is described by Eq.4.A.26. Combining this with Eq.4.A.3 which describes the household's choice for $a_t\delta_t$, we obtain the formulation for optimal road taxes:

$$T_{t,1} = -\frac{U_{M_t}}{U_{c_t}}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial(a_{t-1}\delta_{t-1})}\tilde{\Omega}\Delta_{M_t} - \frac{U_{N_t}}{U_{c_t}}\frac{\partial m_{t,1}}{\partial(a_{t-1}\delta_{t-1})}(1+\tilde{\Omega}\Delta_{N_t}) + \frac{\Phi_t}{U_{c_t}}\frac{\frac{\partial P_t}{\partial x_{t-1}}}{\frac{\partial(a_{t-1}\delta_{t-1})}{\partial x_{t-1}}}, \tag{4.4.36}$$

$$T_{t,2} = -\frac{U_{M_t}}{U_{c_t}}\frac{\partial M_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial(a_{t-2}\delta_{t-2})}\tilde{\Omega}\Delta_{M_t} - \frac{U_{N_t}}{U_{c_t}}\frac{\partial m_{t,2}}{\partial(a_{t-2}\delta_{t-2})}(1+\tilde{\Omega}\Delta_{N_t}) + \frac{\Phi_t}{U_{c_t}}\frac{\frac{\partial P_t}{\partial x_{t-2}}}{\frac{\partial(a_{t-2}\delta_{t-2})}{\partial x_{t-2}}}, \tag{4.4.37}$$

where $x_t = \{k_t^a, l_t^a\}$.

Optimal vehicle purchase tax

Household is subject to vehicle purchase tax when they buy new vehicles. Following the derivation of optimal road tax, we use the first order conditions describing both household's choice and the government's choice for $a_t\delta_t$ (See Eq.4.A.3 and

Eq.4.A.26), to get [8]:

$$\Psi_t \frac{G_{x_t}}{\frac{\partial(a_t\delta_t)}{\partial x_t}} = \lambda_t(1+\tau_t^a)q_t^a.$$

To derive the formulation for the optimal vehicle purchase tax τ_t^a, we turn to the household's problem for firms in the vehicle production sector (see Appendix 4.7.4). With Eq.4.3.29, we get the expression for the optimal vehicle purchase tax:

$$\tau_t^a = \tilde{\Omega}\Delta_{c_t}. \tag{4.4.38}$$

We have obtained the the formulas for the optimal taxes (optimal labour tax, optimal capital income tax, optimal gasoline taxes, optimal road taxes and optimal vehicle purchase tax). While the expressions do not in general provide an explicit formula for the optimal tax rate, it does allow us to draw some conclusions about the tax structure.

4.5 Implications of basic optimal taxes

4.5.1 Optimal labour tax

The formulation of the optimal labour tax is:

$$\tau_t^l = \frac{\tilde{\Omega}(\Delta_{l_t} - \Delta_{c_t})}{1+\tilde{\Omega}\Delta_{l_t}},$$

[8] where we know that

$$\begin{cases} \frac{\partial(a\delta)}{\partial k^a} = \frac{\partial\left(F(k^a,l^a)^2/4\mu\right)}{\partial k^a} = \frac{1}{\mu}\frac{F(k^a,l^a)}{2}F_{k^a}, \\ \frac{\partial(a\delta)}{\partial l^a} = \frac{\partial\left(F(k^a,l^a)^2/4\mu\right)}{\partial l^a} = \frac{1}{\mu}\frac{F(k^a,l^a)}{2}F_{l^a}. \end{cases}$$

where $\tilde{\Omega}$ is the Lagrangian multiplier and is positive in the second-best. The optimal labour tax rate depends on the relation between Δ_{l_t} and Δ_{c_t} (expressed in Eq.4.3.29 and Eq.4.3.30).

Δ_{c_t} is the sum of the elasticities of the marginal utility of consumption with respect to itself, labour and driving service. Similarly, Δ_{l_t} is the sum of elasticities of the marginal utility of labour with respect to itself, consumption and driving service. From the first-order condition (Eq.4.3.30) we know that the denominator part of the optimal labour tax is positive. The unknown part is $\Delta_{l_t} - \Delta_{c_t}$. With Eq.4.3.29 and Eq.4.3.30), we have:

$$\Delta_{l_t} - \Delta_{c_t} = (\frac{U_{ll}l_t}{U_{l_t}} - \frac{U_{lc}l_t}{U_{c_t}}) + (\frac{U_{cl}c_t}{U_{l_t}} - \frac{U_{cc}c_t}{U_{c_t}}) + (1+\gamma)(\frac{U_{Ml}M_t}{U_{l_t}} - \frac{U_{Mc}M_t}{U_{c_t}}). \quad (4.5.39)$$

This formulation suggests a special case which allows us to obtain results which are easier to be interpreted: direct additive utility function. This implies that $U_{ij} = 0$ for $i \neq j$: i.e. $\Delta_{l_t} - \Delta_{c_t}$ can be written as:

$$\Delta_{l_t} - \Delta_{c_t} = \frac{U_{ll}l_t}{U_{l_t}} - \frac{U_{cc}c_t}{U_{c_t}}.$$

The above equation means that the result depends on the elasticity of marginal utility of labour and consumption. Moreover, under the assumption that $U_{ii} < 0$ for labour and consumption, we know that the expression above always stays positive which leads to the result that *when the utility function is directly additive, the optimal labour tax rate is always positive.*

While the strict additive property for utility function is widely applied, it is appealing to consider a general case where marginal utility is dependent of each other. We denote $H_{ki} = -\frac{U_{ki}i}{U_k}$ and it can be interpreted as the elasticity of marginal utility of good k with respect to an increase in good i. We grouped Eq.4.5.39 into three parts with each one describing the complementarity with respect to different utility input. For example, in Eq.4.5.39, the first component measures the labour and consumption's degree of complementarity with labour change. The second component measures consumption and labour's degree of complementarity with consumption and the third measures the driving service and consumption's degree of complementarity with driving service. If one is higher than the other, the good can be said to be more complementary with labour, consumption and driving service respectively. The first two components are negative while the third on is positive. Therefore, the optimal labour tax depends on the interaction of the three components.

4.5.2 Optimal environmental taxes

Our interpretation of the optimal environmental tax formulations base largely on the work done by Sandmo (1975) and Atkinson and Stiglitz (1972). Sandmo (1975) proposes that in the presence of distortionary taxes, the optimal environmental tax is composed of the Pigovian element and the efficiency element. Atkinson and Stiglitz (1972) look into explaining the optimal tax structure using the degree of complementarity to the untaxed goods in the economy which, in our case, is general consumption good.

Optimal gasoline taxes

As shown in Eq.4.4.34 and Eq.4.4.35, the optimal gasoline taxes for new cars and old cars in the steady state are expressed as:

$$\tau^1 = -\frac{U_N}{U_c}\frac{(1+\tilde{\Omega}\Delta_N)}{(1+\tilde{\Omega}\Delta_M)}\frac{m_1}{g_1} + \frac{1}{\delta}\frac{U_Q}{U_c}\frac{(1+\tilde{\Omega}\Delta_Q)}{(1+\tilde{\Omega}\Delta_M)}\frac{\beta}{1-\beta(1-\epsilon)},$$

$$\tau^2 = -\frac{U_N}{U_c}\frac{(1+\tilde{\Omega}\Delta_N)}{(1+\tilde{\Omega}\Delta_M)}\frac{m_2}{g_2} + \frac{1}{\rho\delta}\frac{U_Q}{U_c}\frac{(1+\tilde{\Omega}\Delta_Q)}{(1+\tilde{\Omega}\Delta_M)}\frac{\beta}{1-\beta(1-\epsilon)}.$$

The expression of optimal gasoline taxes have two components. The first part represents the correction towards congestion caused by gasoline usage and it is also proportional to the marginal mileage of travel of gasoline consumption. The second part depicts the damage towards the environment and it depends on the fuel efficiency condition of the vehicles as new cars are more efficient in mitigating pollution (see Eq.4.2.19). In the long run, gasoline taxes are decided by the two opposing components. New cars provide more mileage which leads to heavier congestion but are more efficient in mitigating pollution. On the contrary, old cars cause less congestion but generate more pollutants. In particular:

If we do not consider the congestion externality: the environmental damage caused by gasoline consumption dominates the optimal gasoline tax rates. As new cars are more efficient in mitigating pollution than old cars ($\frac{1}{\delta} > \frac{1}{\rho\delta}$), *the optimal gasoline tax rate is higher for old cars when only pollution externality is considered.*

If we do not consider pollution externality: the congestion externality dominates. As new cars provide more mileage of travel provided that the same amount of gasoline is consumed ($\frac{m_1}{g_1} > \frac{m_2}{g_2}$), *the optimal gasoline tax rate is higher for new* *cars when only congestion externality is considered.*

Given that $\frac{m_1}{g_1} = (a\delta)^\gamma$ and $\frac{m_2}{g_2} = (\rho a\delta)^\gamma$, subtracting the optimal gasoline tax for old cars from the one for new cars gives us:

$$\tau^1 - \tau^2 = \frac{U_N}{U_c}\frac{(1+\tilde{\Omega}\Delta_N)}{(1+\tilde{\Omega}\Delta_M)}[(\rho a\delta)^\gamma - (a\delta)^\gamma] + \frac{1}{\delta}\frac{U_Q}{U_c}\frac{(1+\tilde{\Omega}\Delta_Q)}{(1+\tilde{\Omega}\Delta_M)}(\frac{\rho-1}{\rho}).$$

In the condition where $\rho = 1$, we will have uniform gasoline tax ($\tau^1 = \tau^2$). From the first-order conditions obtained in solving the government's problem (see Eq.4.A.20 and 4.A.21), we know that $1+\tilde{\Omega}\Delta_N$ and $1+\tilde{\Omega}\Delta_M$ are positive. Under the assumption that $0 < \rho < 1$, whether τ^1 is higher than τ^2 depends on two contradicting powers: whether the environmental benefit from using new cars outweighs the negativities from driving too much.

One of the advantages of the model we developed here is that it readily allow us to decompose the tax formula into what might be called an additivity property (as proposed by Sandmo (1975)). Rewriting the optimal gasoline taxes (Eq.4.4.34 and Eq.4.4.35), we can conclude that the optimal tax structure has the following form:

$$\begin{aligned}\tau^1 = & -\frac{U_N}{U_c}\frac{m_1}{g_1} + \frac{1}{\delta}\frac{U_Q}{U_c}\frac{\beta}{1-\beta(1-\epsilon)} \\ & -\frac{U_N}{U_c}\frac{m_1}{g_1}\frac{\tilde{\Omega}(\Delta_N - \Delta_M)}{(1+\tilde{\Omega}\Delta_M)} + \frac{1}{\delta}\frac{U_Q}{U_c}\frac{\beta}{1-\beta(1-\epsilon)}\frac{\tilde{\Omega}(\Delta_Q - \Delta_M)}{(1+\tilde{\Omega}\Delta_M)},\end{aligned} \tag{4.5.40}$$

$$\begin{aligned}\tau^2 = & -\frac{U_N}{U_c}\frac{m_2}{g_2} + \frac{1}{\rho\delta}\frac{U_Q}{U_c}\frac{\beta}{1-\beta(1-\epsilon)} \\ & -\frac{U_N}{U_c}\frac{m_2}{g_2}\frac{\tilde{\Omega}(\Delta_N - \Delta_M)}{(1+\tilde{\Omega}\Delta_M)} + \frac{1}{\rho\delta}\frac{U_Q}{U_c}\frac{\beta}{1-\beta(1-\epsilon)}\frac{\tilde{\Omega}(\Delta_Q - \Delta_M)}{(1+\tilde{\Omega}\Delta_M)}.\end{aligned} \tag{4.5.41}$$

Equations 4.5.40 and 4.5.41 indicate how the presence of the distortionary taxes affects the optimal gasoline tax rates. The first part of the expressions states the special case of a first-best world without the distortionary taxes, where the taxes compensate the marginal environmental damage (pollution and congestion) caused by gasoline consumption. This is the Pigovian tax rate. The second part of the expressions reveal how the presence of distortionary taxes requires a modification to the Pigovian principle.

Optimal road taxes

From Eq.4.4.36 and Eq.4.4.37, we look at the optimal road taxes in the steady state:

$$T_1 = -\frac{U_M}{U_c} M^{1-\sigma} \frac{\gamma m_1^\sigma}{a\delta} \tilde{\Omega} \Delta_M - \frac{U_N}{U_c} \frac{\gamma m_1}{a\delta} (1 + \tilde{\Omega} \Delta_N) + \frac{\Phi}{U_c} \frac{\frac{\partial P}{\partial x}}{\frac{\partial (a\delta)}{\partial x}}, \tag{4.5.42}$$

$$T_2 = -\frac{U_M}{U_c} M^{1-\sigma} \frac{\gamma m_2^\sigma}{a\delta} \tilde{\Omega} \Delta_M - \frac{U_N}{U_c} \frac{\gamma m_2}{a\delta} (1 + \tilde{\Omega} \Delta_N) + \frac{\Phi}{U_c} \frac{\frac{\partial P}{\partial x}}{\frac{\partial (a\delta)}{\partial x}}. \tag{4.5.43}$$

We can see that the road taxes depend on two different parts. The first part represents the driving service provided by owning the vehicles. The second and third parts denote the negative externalities generated by the vehicles.

Following the previous approach, we rewrite the expressions of the optimal road taxes as:

$$\begin{aligned} T_1 = & -\frac{U_N}{U_c} \frac{\gamma m_1}{a\delta} + \frac{\Psi}{U_c} \frac{\frac{\partial P}{\partial x}}{\frac{\partial (a\delta)}{\partial x}} \\ & + \tilde{\Omega} \left[-\frac{U_M}{U_c} M^{1-\sigma} \frac{\gamma m_1^\sigma}{a\delta} \Delta_M - \frac{U_N}{U_c} \Delta_N \right], \end{aligned} \tag{4.5.44}$$

$$\begin{aligned} T_2 = & -\frac{U_N}{U_c} \frac{\gamma m_2}{a\delta} + \frac{\Psi}{U_c} \frac{\frac{\partial P}{\partial x}}{\frac{\partial (a\delta)}{\partial x}} \\ & + \tilde{\Omega} \left[-\frac{U_M}{U_c} M^{1-\sigma} \frac{\gamma m_2^\sigma}{a\delta} \Delta_M - \frac{U_N}{U_c} \Delta_N \right]. \end{aligned} \tag{4.5.45}$$

Similarly, the first part measures the first-best scenario when no distortionary taxes are present. The second part depicts the impact from distortionary taxes. Looking at the difference between T_1 and T_2:

$$T_1 - T_2 = \frac{U_M}{U_c} M^{1-\sigma} \tilde{\Omega} \Delta_M \left[\frac{\gamma (m_2^\sigma - m_1^\sigma)}{a\delta} \right] + \frac{U_N}{U_c} (1 + \tilde{\Omega} \Delta_N) \left[\frac{\gamma (m_2 - m_1)}{a\delta} \right].$$

We will have uniform road taxes ($T_1 = T_2$) when new cars and old cars provide the same mileage of travel ($m_1 = m_2$).

Based on the formulation of the optimal gasoline taxes, another conclusion could be drawn: if congestion is less important to the household than environmental quality, gasoline tax for old cars will be higher than for new ones ($\tau^2 > \tau^1$), which implies that mileage of travel for new cars will be higher than old cars ($m_1 > m_2$). We thus know that the optimal road tax is higher for old cars than new ones ($T_2 > T_1$).

4.6 Conclusion

This chapter examines the optimal environmental tax structure (gasoline taxes and road taxes) in the presence of distortionary taxes and vehicles of different vintages in a dynamic general equilibrium model. Our findings contribute to the literature in several folds. First, we introduce more than one environmental tax to address different externalities caused by vehicle driving. We show that the optimal environmental taxes are related to each other in equilibrium. Second, we find that the additive property between the Pigovian element and the efficiency element proposed by Sandmo (1975) is presented in our model. To which direction the presence of distortionary taxes affect the optimal environmental taxes needs to be investigated further by numerical analysis. Third, we find that the optimal environmental are composed of two opposing factors caused by gasoline consumption and the tax rates are determined by the household's preferences towards the environmental factors. We also applied the approach developed by Atkinson and Stiglitz (1972) to illustrate the optimal tax formulation using the degree of complementartity to general consumption goods.

4.7 Appendix

4.7.1 First-order conditions to the household's problem

Solving the maximization problem, we obtain that:

$$\frac{U_{c_t}}{U_{c_{t+1}}} = \beta \left[1 - \epsilon + (1 - \tau^k_{t+1}) r_{t+1}\right], \tag{4.A.1}$$

$$\frac{\partial U_t}{\partial M_t} \frac{\partial M_t}{\partial m_{t,i}} \frac{\partial m_{t,i}}{\partial g_{t,i}} = U_{c_t}(p_t + \tau^i_t), \qquad i = 1, 2 \tag{4.A.2}$$

$$\beta \left[\frac{\partial U_{t+1}}{\partial M_{t+1}} \frac{\partial M_{t+1}}{\partial m_{t+1,1}} \frac{\partial m_{t+1,1}}{\partial (a_t\delta_t)} - \lambda_{t+1} T_{t+1,1}\right] + \beta^2 \left[\frac{\partial U_{t+2}}{\partial M_{t+2}} \frac{\partial M_{t+2}}{\partial m_{t+2,2}} \frac{\partial m_{t+2,2}}{\partial (a_t\delta_t)} - \lambda_{t+2} T_{t+2,2}\right]$$
$$= \lambda_t (1 + \tau^a_t) q^a_t, \tag{4.A.3}$$

$$U_{1-l_t} = U_{c_t}(1 - \tau^l_t) w_t. \tag{4.A.4}$$

4.7.2 Derivation of the implementability constraint

The budget constraint could be rearranged to:

$$\lambda_t k_{t+1} = \lambda_t \left[1 - \epsilon + (1 - \tau^k_t) r_t\right] k_t + \lambda_t \big[\pi^a_t + \pi^g_t + (1 - \tau^l_t) w_t l_t$$
$$- (p_t + \tau^1_t) g_{t,1} - (p_t + \tau^2_t) g_{t,2} - c_t - (1 + \tau^a_t) q^a_t (a_t\delta_t) - T_{t,1}(a_{t-1}\delta_{t-1}) - T_{t,2}(a_{t-2}\delta_{t-2})\big].$$

It can be expressed as:

$$\lambda_t k_{t+1} = \lambda_t R_t k_t + \lambda_t \{...\}_t, \tag{4.A.5}$$

where $R_t = [1 - \epsilon + (1 - \tau^k_t) r_t]$. The above expression equals:

$$\beta \lambda_t k_{t+1} = \lambda_{t-1} k_t + \beta \lambda_t \{...\}_t. \tag{4.A.6}$$

When time goes to infinity, the value of capital should be zero, which gives us:

$$0 = \lambda_{-1} k_0 + \sum_{t=0}^{\infty} \beta^t \lambda_t \{...\}_t. \tag{4.A.7}$$

Given that the Lagrangian multiplier equals the marginal utility of consumption and that profits are zero when the technology for both production sectors are constant-return-to-scale:

$$0 = \lambda_{-1}k_0 + \sum_{t=0}^{\infty} \beta^t \Big[U_{c_t}(1-\tau_t^l)w_t l_t - U_{c_t}(p_t+\tau_t^1)g_{t,1} - U_{c_t}(p_t+\tau_t^2)g_{t,2} - U_{c_t}c_t$$
$$-U_{c_t}(1+\tau_t^a)q_t^a(a_t\delta_t) - U_{c_t}T_{t,1}(a_{t-1}\delta_{t-1}) - U_{c_t}T_{t,2}(a_{t-2}\delta_{t-2}) \Big]. \tag{4.A.8}$$

Using the first-order conditions, we substitute the taxes with its corresponding real terms:

$$0 = \lambda_{-1}k_0 + \sum_{t=0}^{\infty} \beta^t \Big[U_{1-l_t} l_t - \frac{\partial U_t}{\partial M_t}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}} g_{t,1} - \frac{\partial U_t}{\partial M_t}\frac{\partial M_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial g_{t,2}} g_{t,2} - U_{c_t}c_t$$
$$-U_{c_t}(1+\tau_t^a)q_t^a(a_t\delta_t) - U_{c_t}T_{t,1}(a_{t-1}\delta_{t-1}) - U_{c_t}T_{t,2}(a_{t-2}\delta_{t-2}) \Big]. \tag{4.A.9}$$

Given that:

$$U_{c_t}(1+\tau_t^a)q_t^a(a_t\delta_t) = \beta a_t\delta_t \frac{\partial U_{t+1}}{\partial M_{t+1}}\frac{\partial M_{t+1}}{\partial m_{t+1,1}}\frac{\partial m_{t+1,1}}{\partial (a_t\delta_t)} + \beta^2 a_t\delta_t \frac{\partial U_{t+2}}{\partial M_{t+2}}\frac{\partial M_{t+2}}{\partial m_{t+2,2}}\frac{\partial m_{t+2,2}}{\partial (a_t\delta_t)}$$
$$-\beta a_t\delta_t U_{c_{t+1}} T_{t+1,1} - \beta^2 a_t\delta_t U_{c_{t+2}} T_{t+2,2}, \tag{4.A.10}$$

the three remaining parts in the implementability constraint become:

$$-\big[U_{c_t}(1+\tau_t^a)q_t^a(a_t\delta_t) + U_{c_t}T_{t,1}(a_{t-1}\delta_{t-1}) + U_{c_t}T_{t,2}(a_{t-2}\delta_{t-2})\big] =$$
$$-\Big(\beta a_t\delta_t \frac{\partial U_{t+1}}{\partial M_{t+1}}\frac{\partial M_{t+1}}{\partial m_{t+1,1}}\frac{\partial m_{t+1,1}}{\partial (a_t\delta_t)} + \beta^2 a_t\delta_t \frac{\partial U_{t+2}}{\partial M_{t+2}}\frac{\partial M_{t+2}}{\partial m_{t+2,2}}\frac{\partial m_{t+2,2}}{\partial (a_t\delta_t)}\Big)$$
$$+\big[\beta a_t\delta_t U_{c_{t+1}}T_{t+1,1} + \beta^2 a_t\delta_t U_{c_{t+2}}T_{t+2,2} - U_{c_t}T_{t,1}(a_{t-1}\delta_{t-1}) - U_{c_t}T_{t,2}(a_{t-2}\delta_{t-2})\big]. \tag{4.A.11}$$

We then expand the equation above into different time periods:

$$
\begin{aligned}
&...\\
t=&0 \qquad \beta a_0\delta_0 U_{c_1}T_{1,1}+\beta^2 a_0\delta_0 U_{c_2}T_{2,2}-U_{c_0}T_{0,1}(a_{-1}\delta_{-1})-U_{c_0}T_{0,2}(a_{-2}\delta_{-2}),\\
t=&1 \qquad \beta^2 a_1\delta_1 U_{c_2}T_{2,1}+\beta^3 a_1\delta_1 U_{c_3}T_{3,2}-\beta U_{c_1}T_{1,1}(a_0\delta_0)-\beta U_{c_1}T_{1,2}(a_{-1}\delta_{-1}),\\
t=&2 \qquad \beta^3 a_2\delta_2 U_{c_3}T_{3,1}+\beta^4 a_2\delta_2 U_{c_4}T_{4,2}-\beta^2 U_{c_2}T_{2,1}(a_1\delta_1)-\beta^2 U_{c_2}T_{2,2}(a_0\delta_0),\\
t=&3 \qquad \beta^4 a_3\delta_3 U_{c_4}T_{4,1}+\beta^5 a_3\delta_3 U_{c_5}T_{5,2}-\beta^3 U_{c_3}T_{3,1}(a_2\delta_2)-\beta^3 U_{c_3}T_{3,2}(a_1\delta_1),\\
&...
\end{aligned}
$$

which all cancel out when added up over time and thus the implementability constraint reads:

$$
\begin{aligned}
0=\lambda_{-1}k_0+\sum_{t=0}^{\infty}\beta^t\Big[&U_{1-l_t}l_t-\frac{\partial U_t}{\partial M_t}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}}g_{t,1}-\frac{\partial U_t}{\partial M_t}\frac{\partial M_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial g_{t,2}}g_{t,2}-U_{c_t}c_t\\
&-\beta a_t\delta_t\frac{\partial U_{t+1}}{\partial M_{t+1}}\frac{\partial M_{t+1}}{\partial m_{t+1,1}}\frac{\partial m_{t+1,1}}{\partial(a_t\delta_t)}-\beta^2 a_t\delta_t\frac{\partial U_{t+2}}{\partial M_{t+2}}\frac{\partial M_{t+2}}{\partial m_{t+2,2}}\frac{\partial m_{t+2,2}}{\partial(a_t\delta_t)}\Big].
\end{aligned}
\tag{4.A.12}
$$

We can simplify the expression through Eq.4.2.15, Eq.4.2.16 and Eq.4.2.17:

$$\frac{\partial U_t}{\partial M_t}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}}g_{t,1}+\frac{\partial U_t}{\partial M_t}\frac{\partial M_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial g_{t,2}}g_{t,2}=U_{M_t}M_t, \tag{4.A.13}$$

$$\beta a_t\delta_t U_{M_{t+1}}\frac{\partial M_{t+1}}{\partial m_{t+1,1}}\frac{\partial m_{t+1,1}}{\partial(a_t\delta_t)}=\beta\gamma U_{M_{t+1}}M_{t+1}^{1-\sigma}m_{t+1,1}^{\sigma}, \tag{4.A.14}$$

$$\beta^2 a_t\delta_t U_{M_{t+2}}\frac{\partial M_{t+2}}{\partial m_{t+2,2}}\frac{\partial m_{t+2,2}}{\partial(a_t\delta_t)}=\beta^2\gamma U_{M_{t+2}}M_{t+2}^{1-\sigma}m_{t+2,2}^{\sigma}. \tag{4.A.15}$$

We expand the final two expressions (Eq.4.A.14 and 4.A.15) in time:

$$
\begin{aligned}
&...\\
t =&0 : \beta\gamma U_{M_1} M_1^{1-\sigma} m_{1,1}^{\sigma} + \beta^2 \gamma U_{M_2} M_2^{1-\sigma} m_{2,2}^{\sigma},\\
t =&1 : \beta^2\gamma U_{M_2} M_2^{1-\sigma} m_{2,1}^{\sigma} + \beta^3 \gamma U_{M_3} M_3^{1-\sigma} m_{3,2}^{\sigma},\\
t =&2 : \beta^3\gamma U_{M_3} M_3^{1-\sigma} m_{3,1}^{\sigma} + \beta^4 \gamma U_{M_4} M_4^{1-\sigma} m_{4,2}^{\sigma},\\
&...
\end{aligned}
$$

After summation, we obtain:

$$
\sum_{t=0}^{\infty} \beta^t \big[\beta a_t \delta_t U_{M_{t+1}} \frac{\partial M_{t+1}}{\partial m_{t+1,1}} \frac{\partial m_{t+1,1}}{\partial (a_t\delta_t)} + \beta^2 a_t \delta_t U_{M_{t+2}} \frac{\partial M_{t+2}}{\partial m_{t+2,2}} \frac{\partial m_{t+2,2}}{\partial (a_t\delta_t)} \big] = \sum_{t=0}^{\infty} \gamma \beta^{t+1} U_{M_{t+1}} M_{t+1}. \tag{4.A.16}
$$

4.7.3 First-order conditions to the government's problem

Solving the Lagrangian problem, we get:

$$
c_t : \beta^t \gamma \tilde{\Omega} U_{M_t,c_t} M_t + \beta^t \Big[U_{c_t} + \tilde{\Omega}(U_{l_t,c_t} l_t + U_{M_t,c_t} M_t + U_{c_t,c_t} c_t + U_{c_t}\big) - \Psi_t\Big] = 0, \tag{4.A.17}
$$

$$
l_t : \beta^t \gamma \tilde{\Omega} U_{M_t,l_t} M_t + \beta^t \big[U_{l_t} + \tilde{\Omega}(U_{l_t} + U_{l_t,l_t} l_t + U_{M_t,l_t} M_t + U_{c_t,l_t} c_t) + \Psi_t G_{l_t^g}\big] = 0, \tag{4.A.18}
$$

$$
k_t : \Psi_{t-1} = \beta \Psi_t (G_{k_t^g} + 1 - \epsilon_k). \tag{4.A.19}
$$

Gasoline consumption $g_{t,i}$ are involved in the formulation of $m_{t,i}$, N_t and P_t, we express the decision making process in time scale:

$$
t-1 : \beta^t \tilde{\Omega} \gamma U_{M_t} M_t,
$$

$$
t : \beta^t \big[U(c_t, M_t, 1-l_t, N_t, Q_t) + \tilde{\Omega}(U_{l_t} l_t + U_{M_t} M_t + U_{c_t} c_t + \gamma \beta U_{M_{t+1}} M_{t+1}) - \Phi_t P_t\big].
$$

Therefore, we obtain the first order conditions:

$$\frac{\partial \mathcal{L}^g}{\partial M_t} = U_{M_t}\Big\{1+\tilde{\Omega}\,\overbrace{\big[(1+\gamma)+\frac{(1+\gamma)U_{MM}M_t}{U_{M_t}}+\frac{U_{lM}l_t}{U_{M_t}}+\frac{U_{cM}c_t}{U_{M_t}}\big]}^{\Delta_{M_t}}\Big\}, \tag{4.A.20}$$

$$\frac{\partial \mathcal{L}^g}{\partial N_t} = U_{N_t}\{1+\tilde{\Omega}\,\overbrace{\big[\frac{(1+\gamma)U_{MN}M_t}{U_{N_t}}+\frac{U_{lN}l_t}{U_{N_t}}+\frac{U_{cN}c_t}{U_{N_t}}\big]}^{\Delta_{N_t}}\}, \tag{4.A.21}$$

$$\frac{\partial \mathcal{L}^g}{\partial P_t} = -\,\Phi_t. \tag{4.A.22}$$

Thus for $i=1,2$, the first-order conditions with respect to $g_{t,i}$ are:

$$\frac{\partial \mathcal{L}^g}{\partial g_{t,i}} = U_{M_t}(1+\tilde{\Omega}\Delta_{M_t})M_t^{1-\sigma}m_{t,i}^{\sigma-1}\frac{m_{t,i}}{g_{t,i}}+U_{N_t}(1+\tilde{\Omega}\Delta_{N_t})\frac{m_{t,i}}{g_{t,i}}-\Phi_t\frac{\partial P_t}{\partial g_{t,i}}=\Psi_t p_t, \tag{4.A.23}$$

where $\frac{\partial P_t}{\partial g_{t,1}}=\frac{1}{\delta_{t-1}}$ and $\frac{\partial P_t}{\partial g_{t,2}}=\frac{1}{\rho\delta_{t-2}}$.

Given that a_t and δ_t are functions of k_t^a and δ_t[9], k_t^a, l_t^a and a_t affect $m_{t+1,1}$, $m_{t+2,2}$, P_{t+1} and P_{t+2}. To begin with, the first order condition with respect to the environmental quality Q_t is:

$$\frac{\partial \mathcal{L}^g}{\partial Q_t} = \beta U_{Q_t}\Big[1+\tilde{\Omega}\,\overbrace{\big(\frac{(1+\gamma)U_{MQ}M_t}{U_{Q_t}}+\frac{U_{lQ}l_t}{U_{Q_t}}+\frac{U_{cQ}c_t}{U_{Q_t}}\big)}^{\Delta_{Q_t}}\Big]+\beta\Phi_t(1-\epsilon)-\Phi_{t-1}=0. \tag{4.A.24}$$

The above expression therefore gives us the steady state value of the Lagrangian multiplier Φ_t:

$$\Phi=\frac{\beta U_Q(1+\tilde{\Omega}\Delta_Q)}{1-\beta(1-\epsilon)}. \tag{4.A.25}$$

[9] To produce the optimal amount of $a_t\delta_t$, firms solve the maximization problem max $\delta[F(k_t^a,l_t^a)-\mu\delta]$ with respect to δ which gives $\delta=\frac{F(k_t^a,l_t^a)}{2\mu}$ and $a=\frac{F(k_t^a,l_t^a)}{2}$. Thus we can treat both δ_t and a_t as functions of k_t^a and l_t^a.

For $x_t = \{k_t^a, l_t^a\}$:

$$\frac{\partial \mathcal{L}^g}{\partial x_t} = -\Psi_t G_{x_t} + \beta U_{M_{t+1}} \Big\{ 1 + \tilde{\Omega} \overbrace{\Big[(1+\gamma)(1+\frac{U_{MM}M_{t+1}}{U_{M_{t+1}}}) + \frac{U_{lM}l_{t+1}}{U_{M_{t+1}}} + \frac{U_{cM}c_{t+1}}{U_{M_{t+1}}}\Big]}^{\Delta_{M_{t+1}}} \Big\} \frac{\partial M_{t+1}}{\partial x_t}$$

$$+ \beta^2 U_{M_{t+2}} \Big\{ 1 + \tilde{\Omega} \overbrace{\Big[(1+\gamma)(1+\frac{U_{MM}M_{t+2}}{U_{M_{t+2}}}) + \frac{U_{lM}l_{t+2}}{U_{M_{t+2}}} + \frac{U_{cM}c_{t+2}}{U_{M_{t+2}}}\Big]}^{\Delta_{M_{t+2}}} \Big\} \frac{\partial M_{t+2}}{\partial x_t}$$

$$+ \beta U_{N_{t+1}} \Big\{ 1 + \tilde{\Omega} \overbrace{\Big[(1+\gamma)\frac{U_{MN}M_{t+1}}{U_{N_{t+1}}} + \frac{U_{lN}l_{t+1}}{U_{N_{t+1}}} + \frac{U_{cN}c_{t+1}}{U_{N_{t+1}}}\Big]}^{\Delta_{N_{t+1}}} \Big\} \frac{\partial N_{t+1}}{\partial x_t}$$

$$+ \beta^2 U_{N_{t+2}} \Big\{ 1 + \tilde{\Omega} \overbrace{\Big[(1+\gamma)\frac{U_{MN}M_{t+2}}{U_{N_{t+2}}} + \frac{U_{lN}l_{t+2}}{U_{N_{t+2}}} + \frac{U_{cN}c_{t+2}}{U_{N_{t+2}}}\Big]}^{\Delta_{N_{t+2}}} \Big\} \frac{\partial N_{t+2}}{\partial x_t}$$

$$- \beta \Phi_{t+1} \frac{\partial P_{t+1}}{\partial x_t} - \beta^2 \Phi_{t+2} \frac{\partial P_{t+2}}{\partial x_t} = 0.$$

And we know that:

$$\begin{aligned}
\frac{\partial M_{t+1}}{\partial x_t} &= \frac{\partial M_{t+1}}{\partial m_{t+1,1}} \frac{\partial m_{t+1,1}}{\partial x_t} = M_{t+1}^{1-\sigma} m_{t+1,1}^{\sigma} \gamma \frac{F_{x_t}}{a_t}, \\
\frac{\partial M_{t+2}}{\partial x_t} &= \frac{\partial M_{t+2}}{\partial m_{t+2,2}} \frac{\partial m_{t+2,2}}{\partial x_t} = M_{t+2}^{1-\sigma} m_{t+2,2}^{\sigma} \gamma \frac{F_{x_t}}{a_t}, \\
\frac{\partial N_{t+1}}{\partial x_t} &= \gamma m_{t+1,1} \frac{F_{x_t}}{a}, \\
\frac{\partial N_{t+2}}{\partial x_t} &= \frac{\partial m_{t+2,2}}{\partial x_t} = \gamma m_{t+2,2} \frac{F_{x_t}}{a}, \\
\frac{\partial P_{t+1}}{\partial x_t} &= - \frac{g_{t+1,1}}{\delta_t^2} \frac{F_{x_t}}{2\mu}, \\
\frac{\partial P_{t+2}}{\partial x_t} &= - \frac{g_{t+2,2}}{\rho \delta_t^2} \frac{F_{x_t}}{2\mu}.
\end{aligned}$$

We can then simply the expression for the first order condition with respect to x_t to:

$$\frac{\partial \mathcal{L}^g}{\partial x_t} = -\Psi_t G_{x_t} + \beta U_{M_{t+1}} (1 + \tilde{\Omega} \Delta_{M_{t+1}}) \frac{\partial M_{t+1}}{\partial x_t} + \beta^2 U_{M_{t+2}} (1 + \tilde{\Omega} \Delta_{M_{t+2}}) \frac{\partial M_{t+2}}{\partial x_t} +$$
$$\beta U_{N_{t+1}} (1+\tilde{\Omega}\Delta_{N_{t+1}}) \frac{\partial N_{t+1}}{\partial x_t} + \beta^2 U_{N_{t+2}} (1+\tilde{\Omega}\Delta_{N_{t+2}}) \frac{\partial N_{t+2}}{\partial x_t} - \beta \Phi_{t+1} \frac{\partial P_{t+1}}{\partial x_t} - \beta^2 \Phi_{t+2} \frac{\partial P_{t+2}}{\partial x_t} = 0.$$

We rearrange the above expression using the results we just obtained, we get the first order condition with respect to x_t:

$$\begin{aligned}\frac{\partial \mathcal{L}^g}{\partial x_t} &= -\Psi_t G_{x_t} \\ &+ \beta\big[U_{M_{t+1}}(1+\tilde{\Omega}\Delta_{M_{t+1}})M_{t+1}^{1-\sigma}m_{t+1,1}^{\sigma} + \beta U_{M_{t+2}}(1+\tilde{\Omega}\Delta_{M_{t+2}})M_{t+2}^{1-\sigma}m_{t+2,2}^{\sigma}\big]\gamma\frac{F_{x_t}}{a_t} \\ &+ \beta\big[U_{N_{t+1}}(1+\tilde{\Omega}\Delta_{N_{t+1}})m_{t+1,1} + \beta U_{N_{t+2}}(1+\tilde{\Omega}\Delta_{N_{t+2}})m_{t+2,2}\big]\gamma\frac{F_{x_t}}{a_t} \\ &+ \beta\Phi_{t+1}\frac{g_{t+1,1}}{\delta_t}\frac{F_{x_t}}{F} + \beta^2\Phi_{t+2}\frac{g_{t+2,2}}{\rho\delta_t}\frac{F_{x_t}}{F} = 0.\end{aligned} \tag{4.A.26}$$

4.7.4 Derivation of the optimal vehicle purchase tax

We have showed that the optimal resource allocation for a and δ are: $\delta = \frac{F(k_t^a, l_t^a)}{2\mu}$ and $a = \frac{F(k_t^a, l_t^a)}{2}$, thus in the steady state, the profit maximizing problem for firms can be written as:

$$\max_{k_t^a, l_t^a} \frac{q^a}{\mu}\Big(\frac{F(k^a, l^a)}{2}\Big)^2 - rk^a - wl^a. \tag{4.A.27}$$

The first-order conditions read:

$$\begin{aligned}\frac{q^a}{\mu}\frac{F(k^a, l^a)}{2}F_{k^a} - r = 0, \\ \frac{q^a}{\mu}\frac{F(k^a, l^a)}{2}F_{l^a} - w = 0.\end{aligned}$$

Therefore, we get the expression for the optimal vehicle purchase tax:

$$\Psi_t \frac{G_{x_t}}{\frac{\partial(a_t\delta_t)}{\partial x_t}} = \Psi_t \frac{G_{x_t}}{\frac{1}{\mu}\frac{F(k^a, l^a)}{2}F_{x_t}} = \Psi_t q^a \frac{G_{x_t}}{\frac{q^a}{\mu}\frac{F(k^a, l^a)}{2}F_{x_t}} = \Psi_t q^a. \tag{4.A.28}$$

Chapter 5

Conclusions and future works

This thesis explores the relationship between public policies and vehicle driving from three different aspects: the mechanisms of how public policies affect vehicle driving and the economy; the optimal environmental tax structure in a first-best scenario; and how the presence of distortionary taxes affect the optimal environmental tax structure.

5.1 Conclusions

In Chapter 2, we develop a dynamic general equilibrium infinite-horizon model with physical capital and vehicles, where vehicles are of two vintages (new and old), and investigate the impact of two policy options (gasoline taxes and clean technology subsidies) on driving behavior, vehicle production, fuel consumption, environmental quality and social welfare. We contribute to the literature in several folds. To begin with, differently from Parry and Small (2005), we develop a framework where dynamic relationships are present to capture the long-run nature of pollution and capital accumulation. A dynamic model is useful to interpret pollution issues as those generally accumulate over time and pollution also affects the environmental quality over time. Secondly, we extend Wei (2013)'s model where she uses vehicle capital gasoline consumption ratio as the only production input (Leontief production possibility). Instead, we adopt capital heterogeneity to model vintage vehicles in that it generates mileages of travel given any amount of gasoline pumped in. Leontief production possibilities thus do not match with this feature. Furthermore, we offer a novel way of modelling vehicle capital and fuel efficiency. Previous literature has assumed that all components of vehicle capital are indistinguishably linked to fuel efficiency. We expand this framework to model two distinct attributes of the vehicle capital to capture the impact from policies targeting at different aspect of the production process. We first find that, in terms of driving choices, the households purchase more fuel for new cars than old cars and households prefer to use new cars more often than old cars. Our simulation based on the U.S. economy show that fuel consumption and pollution levels decrease under all the policy options. However, they have distinctively different distributional impact. Levying gasoline taxes do not improve the overall fuel efficiency (miles per gallon) of the vehicles and also change the production side only slightly. It alleviates pollution which in turn enhances the environmental quality and eventually improves the social welfare. Providing subsidies to clean technology, instead, leads to more resources being allocated to the production of fuel-efficient and cleaner engines, which results in higher capital accumulation and labour supply in the vehicle production sector. As subsidy rate increases, social welfare first improves and then plunges when production inefficiencies kick in.

In Chapter 3, we derive the optimal steady-state first best environmental tax structure in the presence of (i) different vintage vehicles (new and old); (ii) pollution and congestion externalities caused by vehicle driving. We contribute to the theoretical literature in several ways. First of all, to fully tackle the external cost generated by vehicle driving, we examine the first best environmental taxes to address pollution and congestion externalities separately. Previous literature focused mainly on using one instrument to address all the externalities caused by vehicle driving. Our model allows us to capture the interrelation between different environmental taxes and see how it affects the optimal tax structure. Analytical results show that the first best optimal gasoline taxes consist of two opposing parts caused by gasoline consumption: marginal cost of pollution and marginal cost of congestion. New cars generate less pollution but contribute more to the mileage of travel, which leads to heavier congestion. Thus, the optimal gasoline taxes for different types of vehicles depend on the two opposing factors. Optimal road taxes target at congestion externality which is related to the vehicle fuel efficiency level. In the steady states, households prefer to drive new cars more often which implies higher mileage of travel, and therefore the road tax is higher for new cars than old cars. We further derive the uniform gasoline tax and the formula takes the form of weighted average of gasoline taxes for new cars and old cars. Second, we calibrate our model based on the U.S. economy and show that the optimal environmental taxes depend on the households' preference for environmental factors. In the presence of congestion externality, optimal gasoline tax for old vehicles is higher than for new vehicles when households start to value environment. And that shows the case when the marginal cost of pollution outweighs the marginal cost of congestion. Households are better off under optimal fuel tax than uniform fuel tax but not to a substantial extent.

In Chapter 4, we look into the optimal environmental tax (gasoline taxes and road taxes) structure for vehicles of different vintages (new cars and old cars) in the presence of other distortionary taxes and externalities caused by vehicle driving (pollution and congestion). We extend the literature by looking at different environmental taxes and how they relate to each other in a second best scenario. The optimal environmental tax formulas present the additive property between the Pigovian element and the efficient element proposed by Sandmo (1975). The presence of distortionary taxes causes the optimal environmental taxes to deviate from the Pigovian standards and the deviation depends on household's preference for the environmental factors. To further examine the results of our optimal environmental taxes, we apply the approach proposed by Atkinson and Stiglitz (1972) and look into how optimal tax structure can be explained by the degree of complementarity to normal consumption goods. We also find that the formulation of the optimal environmental taxes depend on two opposing factors. Optimal gasoline taxes depend on the marginal cost of pollution and marginal cost of congestion while optimal road taxes depend on the marginal benefit from owning vehicle and the marginal cost of externalities from using the vehicles. And whether one factor outweighs the other depends on utility inputs' degree of complementarity to the normal consumption goods.

5.2 Future works

There are still various aspects relating to the roles of public policies in the regime of vehicle driving that haven't been the focuses of this thesis.

First of all, we do not specifically examine the dynamic properties of the policy impact because we focus more on the long-run changes. The difficulty of examining the dynamic properties is that vehicles are modelled as a special type of capital which are different across vintages and last for only two periods of time. Moreover the focus of our model is to illustrate the long-run impact of the policies rather than the inter-temporal changes. However, dynamic properties will enable us to see the short-run impact of policies and how policy shocks would change the economy and to what extent.

Secondly, we do not allow the households to have the freedom when it comes to vehicle purchase decision making. At each time of period, we assume that the households invest in new vehicles of the latest technology and scrape the old ones after two periods of usage. It would be interesting to develop a heterogeneous agent model and allow different households to choose from purchasing new vehicles, keeping using their old ones or scraping old vehicles.

Thirdly, we treat mileage of travel as a type of service and do not model the time effect into the decision making process for households. Given the fact that roads have become more congested, time spent on roads is a growing concern for households. How to model the time effect into the model would be an interesting extension.

5.3 Concluding marks

To conclude, the main contribution of this thesis are: (1) providing a thorough picture on the fundamental mechanisms to explain how different public policies affect vehicle driving, the economy, environment and social welfare; (2) deriving the optimal environmental tax structure in the first-best scenario and examining the interrelation between optimal environmental taxes; and (3) constructing a theoretical framework to understand the optimal environmental tax structure in the presence of other distortionary taxes. The model developed in this thesis could be applied for most countries where tax and subsidy schemes are possible. In numerical experiments, our calibration is based on the U.S. economy but could be done for other countries as well. The additional consideration to enrich this thesis include adding in dynamic properties and allowing households more freedom to vehicle-related decision making. This thesis focuses on the theoretical aspects of public policies and vehicle driving, we thus leave empirical analysis to future works.

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